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Kosuke Aoki, Ko Munakata, and Nao Sudo

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Abstract

Banks in developed countries share a common concern that prolonged low nominal interest rates may pose a threat to their business, as the level of nominal interest rates is often positively correlated with bank profits in the data. It is not well understood, however, how low nominal interest rates impact bank profits and what they imply for banking stability. To address these issues, this study theoretically explores how the level of nominal interest rates affects bank profits and banking stability in the long run by extending a model of bank runs constructed by Gertler and Kiyotaki (American Economic Review, 2015). The model, calibrated to Japan and other developed countries, makes three predictions: (1) low interest rates do indeed reduce bank profits by compressing the deposit spread; (2) due to the presence of the effective lower bound of the policy rate and a slow recovery of bank net worth after a run, low interest rates bring the economy closer to a state where a bank run equilibrium can exist; (3) although there are quantitative differences across countries, a decline in nominal interest rates does not necessarily bring the economy to a state with a bank run equilibrium on its own, except for in severe cases where the TFP growth rate or the target inflation rate falls below zero.

Keywords: Prolonged low interest rates; Bank profits; Banking stability
JEL classification: E20, J11

*Professor, University of Tokyo (E-mail: kaoki@e.u-tokyo.ac.jp)
**Associate Director, Research and Statistics Department, Bank of Japan (E-mail: kou.munakata@boj.or.jp)
***Director, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: nao.sudou@boj.or.jp)

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1 Introduction

Even 10 years after the global financial crisis, the short-term nominal interest rate in developed countries remains low. As shown in panel (a) of figure 1, in all the countries presented except for Japan, which has seen the rate close to zero since the mid-1990s, the short-term rate fell to a level around zero during the crisis period and has not yet returned to its pre-crisis level. For example, the interest rate in the U.S. as of March 2019 was 240 basis points, which is 280 basis points lower than the pre-crisis level. In addition, there is reason to believe that low nominal interest rates may continue in the years ahead. This is because, theoretically, the nominal interest rate reflects developments in the natural rate of interest and, according to the secular stagnation hypothesis proposed by Summers (2014, 2015), the natural rate of interest may remain low going forward. Panels (b) and (c) of figure 1 show that developments in the natural rate of interest, or r^*, have generally tracked those of nominal interest rates, showing no signs of rising up to the present.[1] The current and predicted prolonged low nominal interest rates are worrying for banks. This is because the level of nominal interest rates and bank profits are often positively related in the data. Panel (d) of figure 1 shows the contemporaneous correlation between the spread between banks' lending rate and deposit rate, which proxies for the size of bank profits, and the nominal interest rate in G7 countries during the last twenty years. The slope is clearly positive.

In this paper, we explore the implications of prolonged low nominal interest rates for banks. We address the following two questions: (1) Why do nominal interest rates move hand-in-hand with bank profits and (2) how do prolonged low nominal interest rates affect banking stability? These two questions are not novel, having attracted the attention of policy makers as well as scholars repeatedly in recent years.[2] While there has been much

[1] The natural rates of interest shown in panel (b) are all estimated based on the methodology developed in Laubach and Williams (2003). Those shown in panel (c) are of Japan and have been taken from Sudo et al. (2018). They are estimated using various methodologies, including that of Laubach and Williams (2003).

[2] For example, the Committee on the Global Financial System (CGFS, 2018) addresses the same set of questions, by empirically studying bank-level data in developed and developing countries. It concludes that although banks' interest income may be compressed under prolonged low nominal interest rates, banks can undertake a number of adjustments to shield their profitability from the adverse effects of low rates.

empirical work on the first question, as of this writing, empirical studies on the second question and theoretical studies on both questions remain scarce.

We therefore develop a dynamic stochastic general equilibrium (DSGE) model and provide answers to both questions. We choose the model settings so that the model can address two facts that are considered to be important for banks. The first is the tight positive relationship between the deposit spread and the level of nominal interest rates. Figure 2 shows the contemporaneous correlation between the deposit spread and the nominal interest rate for G7 countries. One can see that it is the deposit spread rather than the lending spread that is positively correlated with nominal interest rates. In other words, the decline in bank profits that accompanies a decline in the nominal interest rate is caused by compressed deposit spreads.[3,4] The second is that banks compete with financial services provided outside the banking sector. How low nominal interest rates affect banks depends, therefore, on how differently they affect banks compared to other financial institutions and how financial services provided by banks are replaced by financial services provided by other financial institutions. Panel (a) of figure 3 shows the share of financial assets held by banks in selected G7 countries. It indicates that banks' lending services have already been replaced substantially in Canada and the U.S.[5] Panel (b) of figure 3 shows the stock prices of banks and other financial firms in Japan around noon on the 29th of January 2016, when the introduction of the negative interest rate policy was announced by the Bank of Japan. While the TOPIX index, Japan's aggregate stock price index, and the stock prices of some non-bank financial institutions rose following the news, the stock prices of banks fell, suggesting that there was a perception among market participants that banks and other types of financial firms would react differently to low nominal interest rates.

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[3]This observation agrees with existing studies. In fact, as discussed in the next section, some empirical studies, such as Borio et al. (2015) and Claessens et al. (2018), stress the importance of imperfect pass-through of the short-term nominal interest rate to deposit rates for understanding how low nominal interest rates reduce bank profits. It is also notable, however, that while these studies often propose the lower bound of deposit rates as an explanation for the imperfect pass-through, as panel (b) of figure 2 shows, the positive relationship had already manifested itself during the pre-crisis period.

[4]The positive relationship is still found even when alternative measures for the deposit spread and nominal interest rates are used. See the sensitivity analysis shown in Table 1.

[5]The figures are taken from the Global Shadow Banking Monitoring Report, released by the Financial Stability Board (FSB, 2018).

Our model is built upon the model of bank runs developed by Gertler and Kiyotaki (2015). Banks earn profits by extending credit to firms and providing liquidity services to households via deposits. Bank lending competes with lending extended by non-bank lenders and deposits compete with money. A bank run takes place when households decide to withdraw their deposits. Whether or not households withdraw deposits depends on the expected liquidation price of bank assets in the event of a run, which is endogenously determined by economic fundamentals, including the nominal interest rate. The key difference between our model and that of Gertler and Kiyotaki (2015) is that ours incorporates households' utility gains that arise from liquidity services provided by deposits and money, following Drechsler et al. (2017), and a central bank that adjusts the nominal interest rate with the aim of stabilizing the inflation rate at a target rate.

We calibrate the model using Japanese data. We choose Japan, since it is a country where nominal interest rates have been low for more than twenty years up to now and the effect of prolonged low nominal interest rates on bank profits and banking stability is a pressing concern. We choose values for the model parameters so that the values of the key endogenous variables computed from the model are consistent with the data. These variables include those shown in figures 2 and 3, namely the elasticity of the deposit spread to a change in nominal interest rates and the relative size of the banking sector compared to non-bank lenders. We then study bank profits and the degree of banking stability in economies with different growth rates of total factor productivity (TFP) and different target inflation rates set by the central bank. We study the effects of changes in the two determinants of long-run nominal interest rates rather than the effects of exogenous changes in nominal interest rates as we are interested in the effect of prolonged low nominal interest rates.

Our findings are summarized in three points. Firstly, prolonged low nominal interest rates lower bank profits by compressing the deposit spread. This holds true regardless of which of the two factors, the TFP growth rate or the target rate of inflation, drives the nominal interest rate down. With low nominal interest rates, households demand less deposits and more money as a means of exchange, which, in turn, lowers the price of liquidity services provided by deposits. The compressed deposit spread results in a smaller net interest margin (NIM) and smaller bank net worth.

Secondly, prolonged low nominal interest rates undermine banking stability by driving the economy closer to a state where a bank run equilibrium exists. This occurs through two channels, one that arises from the effective lower bound (ELB) of the policy rate during a run and another that arises from the pace at which bank net worth accumulates after a run. When the nominal interest rate is already low in normal times, then the policy rate is more likely to hit the ELB. When the ELB binds, the liquidation price of bank assets falls more during a run, increasing the incentive for households to withdraw their deposits. In addition, when the nominal interest rate is low in normal times, the same mechanism that compresses bank profits continues to operate and the deposit spread becomes narrower during a run, causing banks to require a longer time to recover following a run. The prospect of a slow recovery for banks after a run weakens the demand for bank assets during a run, which, in turn, reduces the liquidation value of its assets.

Thirdly, low nominal interest rates do not necessarily mean that the economy is in a state with a run equilibrium. Indeed, based on our calibration for Japanese economy, neither a low TFP growth rate nor a low target inflation rate causes a run on its own when taking a positive value.

In addition to the analysis for Japan's economy, we conduct similar simulations for Canada, the U.K., and the U.S. We find that the three findings above hold for these countries as well. There are, however, quantitative differences across countries. In particular, the impact of nominal interest rates on banking stability is less pronounced in Canada and the U.S., where non-bank lenders are considered to play a relatively larger role in the financial system. In fact, our simulation exercises indicate that efficient non-bank lenders enhance banking stability by mitigating the fall in the liquidation price of bank assets during a run and reducing households' incentive to run, although in normal times they reduce bank profits.

The remainder of this paper is organized as follows. Section 2 provides a literature review. Section 3 describes the model and the calibration strategy. Section 4 shows our simulation results. Section 5 conducts international comparisons. Section 6 concludes.

2 Literature Review

Our paper is built upon three strands of the literature. The first strand includes empirical studies on the relationship between the level of nominal interest rates and bank profits, such as Borio et al. (2015), Di Lucido et al. (2017), IMF (2017), and Claessens et al. (2018). These studies commonly report that the two variables are positively correlated and propose incomplete pass-through from short-term nominal interest rates to deposit rates as an explanation for the correlation.[6] For example, using data for 109 large international banks, Borio et al. (2015) document that the relationship between banks' net interest income and the level of the short-term nominal interest rate is positive and non-linear and argue that this observation is consistent with the incomplete pass-through explanation.

The second strand of literature includes theoretical work on how compressed bank profits, due to low nominal interest rates, change the nature of the transmission mechanism of monetary policy. Based on the empirical findings documented in the strand of the literature mentioned above, these studies argue that the accommodative impact of expansionary monetary policy shocks may be attenuated when the policy rate is low. For example, Brunnermeier and Koby (2017) develop a model with banks that are subject to capital and liquidity constraints. They show that when the policy rate is already low, a further cut in the policy rate may hamper rather than boost bank lending, leading to a downturn in output. Along the same lines, Eggertsson et al. (2017) construct a New Keynesian model with banks and show that a cut in the policy rate can be contractionary through a mechanism that is qualitatively similar to that discussed in Brunnermeier and Koby (2017).[7] Our paper is closely related to these studies as it is also motivated by the findings of the first strand of the literature. Our paper differs from them, however, in terms

[6] From a similar but slightly different angle, Drechsler et al. (2017) stress the importance of the market power of banks in the deposit market in explaining the observation that the deposit rates of U.S. banks do not move one-for-one with the federal funds rate.

[7] Heider et al. (2018) empirically study the impact of negative interest rates in the euro area and show that low interest rates do indeed reduce bank lending.

of its focus. In particular, this paper studies the long-run effects of low nominal interest rates rather than those of a short-run innovation to the nominal interest rate and explores not only the effect on bank profits, but also those on banking stability.

The third strand of the literature includes studies on the substitutability between bank loans and other means of financing and studies on the substitutability between bank deposits and money, including Meltzer (1960), Greenspan (1999), Allen and Gale (2000), Gambacorta et al. (2014), Levin et el. (2016), and Drechsler et al. (2017). For example, Meltzer (1960) famously argues that trade credits extended by large firms attenuate the effects of credit rationing on small firms due to monetary tightening. Greenspan (1999) proposes a view called the "Spare Tire Hypothesis," pointing out that financial diversity helps limit the effects of economic shocks, including those of banking crises.[8,9] Using U.S. regional data, Drechsler et al. (2017) focus on the liability side of banks' balance sheets and argue that the substitutability between deposits and money and the substitutability between deposits of a bank and those provided by other banks play important roles in the monetary policy transmission. Our work complements these studies by exploring the implications of substitution among financial services for banking stability, which is an area that has not been studied much thus far.

Our paper is also related to studies on bank runs, in particular Gertler and Kiyotaki (2015). Although our model is built upon Gertler and Kiyotaki (2015), our analysis differs in its focus and model settings. Namely, while Gertler and Kiyotaki (2015) focus on understanding the short-run dynamics of the economy, such as the feedback mechanism of a run to economic activity in normal times, our focus is on the long-run effects of prolonged low nominal interest rates on bank profits and banking stability. For this purpose, we add new ingredients to Gertler and Kiyotaki (2015). The two key ingredients of our model, the liquidity services of deposits and the effective lower bound of the policy rate, are absent in

[8] Along these lines, recent papers by Buchak et al. (2018) and Tang (2018) empirically assess how tightened regulations on banks influence lending activities by Fintech lenders in the residential mortgage and consumer credit markets in the U.S., respectively.

[9] See also Kashyap et al. (1993), who argue that bank and non-bank sources of finance are not perfect substitutes by empirically studying how the supply of bank credit and commercial paper issuance vary in response to changes in monetary policy.

Gertler and Kiyotaki (2015).

3 Model

Our model is built upon those of Gertler and Kiyotaki (2015) and Drechsler et al. (2017). The economy consists of five sectors: households, banks, non-bank lenders, the goods-producing sector, and the government sector.

- Households: A representative household supplies labor to firms and earns wages. It holds its assets in four forms: bank deposits, claims on non-bank lenders, government bonds, and money. It receives monetary returns from these four assets and receives liquidity services from deposits and money holdings.
- Banks: Banks collect deposits from households and provide liquidity services via deposits. They also provide lending services by financing goods-producing firms with deposits and their own net worth. Similar to the model in Gertler and Kiyotaki (2015), banks default when they are unable to repay their deposit obligations to households.
- Non-bank lenders (NBL): NBLs collect funds from households and finance goods-producing firms. In contrast to banks, NBLs do not provide liquidity services.
- Goods-producing sector: The goods-producing sector consists of intermediate goods firms, wholesale goods firms, and final goods firms. Intermediate goods firms hire labor inputs from households and capital inputs from NBLs and banks to produce intermediate goods. Wholesale goods firms produce wholesale goods from intermediate goods. Final goods firms produce final goods from wholesale goods.
- The government sector consists of the government, which collects taxes from households to finance its repayments to bondholders, and the central bank, which adjusts the nominal interest rate so as to stabilize the inflation rate. The central bank is not able to cut the rate to below zero.

Our model differs from that of Gertler and Kiyotaki (2015) and also from its extension in Gertler et al. (2018), in two main ways. Firstly, our model explicitly incorporates households' utility gains from liquidity services provided by bank deposits. Similar to Van den Heuvel (2008) and Drechsler et al. (2017), the deposit spread reflects households' demand for liquidity services. Secondly, our model assumes an effective lower bound of zero for the policy rate. As we discuss below, the zero lower bound limits the effectiveness of monetary easing during a crisis period, which in turn brings about a negative feedback effect on banking stability in normal times.

As we describe below, the economy faces a run depending on economic conditions. For notational convenience, we denote the value of a variable in normal times by, say, S_t, and the value of the same variable during a run by S_t^*. An outline of the model is illustrated in figure 4.

3.1 Households

Setting

A representative household is infinitely lived and gains utility from consumption C_t, real bank deposits $\frac{D_t}{P_t}$, and real money holdings $\frac{M_t}{P_t}$, and disutility from labor L_t, where P_t is the price level. It holds its assets in four forms: bank deposits $\frac{D_t}{P_t}$, capital stock $K_{h,t}$, government bonds $\frac{B_t}{P_t}$, and money $\frac{M_t}{P_t}$. We assume that management of capital stock $K_{h,t}$ is entrusted to NBLs. Households pay NBLs a fee, denoted $F\left(K_{h,t}\right)$, that is a function of the size of the capital stock, and receives nominal return $R_{NB,t}$ from NBLs. The utility maximization problem of the household is given as follows.

$$U_t \equiv \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \left[\log(C_{t+i}) - \chi \frac{L_{t+i}^{1+v}}{1+v} + \omega \Omega \left(\frac{D_{t+i}}{P_{t+i}}, \frac{M_{t+i}}{P_{t+i}} \right) - \tilde{1}_{t+i} \delta \log \left(\frac{D_{t+i-1}}{P_{t+i-1}} \right) \right], \quad (1)$$

where

$$\Omega \left(\frac{D_t}{P_t}, \frac{M_t}{P_t} \right) \equiv \log \left(\left[(\mu \frac{D_t}{P_t})^{\zeta} + (\frac{M_t}{P_t})^{\zeta} \right]^{\frac{1}{\zeta}} \right), \quad (2)$$

subject to the budget constraint

$$C_{h,t} + \frac{D_t + M_t + B_t}{P_t} + Q_t K_{h,t} + F\left(K_{h,t}\right) =$$
$$\frac{L_t W_t + R_{d,t} D_{t-1} + M_{t-1} + R_{GB,t-1} B_{t-1} + R_{NB,t} K_{h,t-1}}{P_t} + Q_t K_{h,t-1} + \Pi_t - T_t.$$

Here $\beta \in (0,1)$ is the subjective discount factor, $\chi > 0$ is the weight on disutility arising from labor, $\upsilon > 0$ is the inverse of the Frisch elasticity of labor supply, and $\omega > 0$ is the weight on utility arising from liquidity services provided by deposits and money holdings. W_t, $R_{d,t}$, and $R_{GB,t}$ are the nominal wage, nominal deposit rate, and nominal interest rate. The deposit rate $R_{d,t}$ takes a contractual rate in normal times, which we denote as $\overline{R}_{d,t-1}$, and takes the value $x_t \overline{R}_{d,t-1}$, for $x_t < 1$, when a run takes place. Hereafter, we refer to the variable x_t as the recovery rate. Q_t is the market price of a unit of capital stock. The term Ω represents utility from liquidity services, where $\mu > 0$ captures the relative size of liquidity services provided by deposits and ζ is the elasticity of substitution between money and deposits. Because deposits and money are considered substitutes, we assume that $\zeta > 0$, similar to Drechsler et al. (2017). Π_t is the sum of transfers from wholesale goods firms and T_t is a lump-sum tax taken by the government. As shown in the last term of equation (1), we assume that households receive disutility from holding deposits D_t in the beginning of a period when a run occurs in addition to the monetary loss that is described later. This disutility represents delays or difficulties associated with transactions that involve deposits D_t during a run and the size of the disutility is governed by the parameter $\delta \geq 0$. $\tilde{1}_{t+i}$ is an indicator function that takes a value of unity if a run occurs in period $t+i$ and zero otherwise, implying that the disutility arises only in the quarter that a run occurs.[10]

[10] We incorporate the parameter δ in order to give a degree of flexibility into the model regarding how much banks take risks when the economy is at a state with a run equilibrium due to low nominal interest rates. In the appendix we study the implications of incorporating this disutility into the model by comparing the model under various values of the parameter δ. We show that the size of this parameter does not affect whether or not the economy is at a state with a run equilibrium. In addition, it does not affect prices and allocations of the economy as far as the economy is not at a state with a run equilibrium.

Household asset allocation

Households allocate their total assets to four types of assets. The first order conditions associated with asset holdings D_t, $K_{h,t}$, B_t, and M_t are given as follows.

$$\frac{1}{C_t} = \beta \mathrm{E}_t \left[\frac{R_{d,t+1}}{\pi_{t+1} C_{t+1}} \right] + \omega \frac{\mu^{\zeta} (\frac{D_t}{P_t})^{\zeta - 1}}{(\mu \frac{D_t}{P_t})^{\zeta} + (\frac{M_t}{P_t})^{\zeta}}, \tag{3}$$

$$\frac{1}{C_t} = \beta \mathrm{E}_t \left[\frac{Q_{t+1} + R_{NB,t+1}/P_{t+1}}{C_{t+1}} \right] \frac{1}{(Q_t + \partial F (K_{h,t}) / \partial K_{h,t})}, \tag{4}$$

$$\frac{1}{C_t} = \beta R_{GB,t} \mathrm{E}_t \left[\frac{1}{\pi_{t+1} C_{t+1}} \right], \tag{5}$$

$$\frac{1}{C_t} = \beta \mathrm{E}_t \left[\frac{1}{\pi_{t+1} C_{t+1}} \right] + \omega \frac{(\frac{M_t}{P_t})^{\zeta - 1}}{(\mu \frac{D_t}{P_t})^{\zeta} + (\frac{M_t}{P_t})^{\zeta}}. \tag{6}$$

Notice that when holding deposits D_t, households receive interest payments with the rate given by the deposit rate $R_{d,t+1}$ and utility gains from liquidity services, as shown in the first and second terms of the right hand side of equation (3), respectively. There is no utility gain from holding claims against NBLs $K_{h,t}$ or government bonds B_t as these assets do not provide liquidity services. When holding money M_t, households receive a return equal to the inverse of the inflation rate π_{t+1}^{-1}, where π_t is the growth rate of goods price P_t, and utility gains from liquidity services, as shown in the first and second terms of the right hand side of equation (6), respectively.

3.2 Goods-producing sector

The goods producing sector consists of three types of producers: intermediate goods producers, wholesale goods producers, and final goods producers. The model settings regarding these producers are standard settings.

Intermediate goods producers

Intermediate goods producers produce intermediate goods y_t and sell them to wholesale goods producers at price $P_{y,t}$. They hire labor L_t from households and borrow capital K_t

from banks and NBLs, taking prices as given. The maximization problem of these firms is given by

$$\max_{y_t,\, K_t,\, L_t} \frac{P_{y,t} y_t}{P_t} - Z_t K_t - \frac{W_t}{P_t} L_t, \tag{7}$$

subject to

$$y_t = A_t \left(K_t\right)^{1-\eta} \left(L_t\right)^{\eta},$$

where Z_t is the marginal product of capital (MPK), A_t is the level of TFP, and $\eta \in [0, 1]$ is the labor share. We further assume that TFP grows at a constant rate z in a deterministic manner:

$$\log A_{t+1} - \log A_t = z.$$

The first order conditions of intermediate goods producers yield the following equalities:

$$\begin{aligned} Z_t &= (1-\eta) A_t \frac{P_{y,t}}{P_t} \left(K_t\right)^{-\eta} \left(L_t\right)^{\eta}, \\ \frac{W_t}{P_t} &= \eta A_t \frac{P_{y,t}}{P_t} \left(K_t\right)^{1-\eta} \left(L_t\right)^{\eta-1}. \end{aligned} \tag{8}$$

Wholesale goods producers and final goods producers

The wholesale goods sector contains a continuum of firms, each producing a differentiated product, indexed by $h \in [0, 1]$, from intermediate goods using a linear production technology given by

$$\tilde{y}_t(h) = y_t(h),$$

where $\tilde{y}_t(h)$ denotes the differentiated wholesale good produced by wholesale goods producer h and $y_t(h)$ is the intermediate good used as an input by producer h. Final goods producers purchase these differentiated goods in a competitive market and produce final goods based on the following constant elasticity of substitution (CES) aggregate technology:

$$Y_t = \left[\int_0^1 \tilde{y}_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{1-\varepsilon}}, \quad \varepsilon > 1$$

where $\varepsilon \in (1, \infty)$ denotes the elasticity of substitution between differentiated wholesale goods. Given this CES technology for final goods, the demand for each differentiated wholesale good $\tilde{y}_t(h)$ is given by the following function of its price $P_t(h)$, the aggregate price index P_t, and the aggregate demand for final goods Y_t:

$$\tilde{y}_t(h) = \left(\frac{P_t(h)}{P_t} \right)^{-\varepsilon} Y_t.$$

Each wholesale goods producer h maximizes its profit by choosing the optimal product price, although it has to pay adjustment costs à la Rotemberg (1982) whenever adjusting its price. Its maximization problem is given by

$$\max_{P_{t+j}(h)} \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \frac{C_t}{C_{t+j}} \left[\begin{array}{l} \left(\frac{P_{t+j}(h)}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j} \\ - \left(\frac{P_{y,t+j}}{P_{t+j}} \right) \left(\frac{P_{t+j}(h)}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \\ - \frac{\kappa}{2} \left(\frac{P_{t+j}(h)}{P_{t+j-1}(h)} - \overline{\pi} \right) \left(\frac{P_{t+j}(h)}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \end{array} \right],$$

where the third term represents the adjustment cost it has to pay when changing the price of its product $P_t(h)$, κ is a parameter that governs the size of the cost, and $\overline{\pi}$ is the target inflation rate set by the central bank.

Assuming a symmetric equilibrium, all of the differentiated goods prices $P_t(h)$ set by wholesale goods producers are identical and the Phillips curve of the economy is given by

$$\begin{array}{l} -\varepsilon \left(1 - \frac{P_{y,t}}{P_t} - 0.5 \left(\pi_t - \overline{\pi} \right)^2 \right) + 1 - \kappa \left(\pi_t - \overline{\pi} \right) \pi_t \\ + \beta \mathrm{E}_t \frac{C_t}{C_{t+1}} \kappa \left(\pi_{t+1} - \overline{\pi} \right) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0. \end{array} \tag{9}$$

3.3 Banks

The bank's problem

The banking sector consists of a large number of banks. Each bank collects deposits $\frac{d_t}{P_t}$ and extends credit $k_{b,t}$ to intermediate goods producers. Each bank receives earnings $Z_t k_{b,t-1}$ from intermediate goods producers, repays deposit obligations $\frac{R_{d,t}}{\pi_t}\frac{d_{t-1}}{P_{t-1}}$ to households, and accumulates net worth n_t by retaining the remaining earnings. Similar to Gertler and Kiyotaki (2015), we assume that banks have a finite expected lifetime. Each bank has an i.i.d. probability $\sigma \in (0,1)$ of surviving until the next period and a probability $1-\sigma$ of exiting. A new bank enters the economy with a transfer from households $A_t w^b$.

The bank's optimization problem is therefore expressed as follows.

$$V_t = \mathrm{E}_t \left[\sum_{i=1}^{\infty} \beta^i \left(1-\sigma\right) \sigma^{i-1} n_{t+i} \right], \tag{10}$$

subject to three constraints of a law of motion for the bank's net worth n_t, a balance sheet constraint, and a capital constraint, as described below.

$$n_t = \left(Z_t + Q_t\right) k_{b,t-1} - \frac{\overline{R}_{d,t-1}}{\pi_t}\frac{d_{t-1}}{P_{t-1}}, \tag{11}$$

$$Q_t k_t^b = \frac{d_t}{P_t} + n_t, \text{ and} \tag{12}$$

$$Q_t k_t^b \leq \phi n_t, \tag{13}$$

where $\phi > 0$ is a parameter that takes a positive value.

Note that the only difference between the model in Gertler and Kiyotaki (2015) and our model regarding the settings of the banking sector is that in our model banks are subject to capital constraint (13) while in Gertler and Kiyotaki (2015) they are subject to the incentive constraint described below.

$$\theta Q_t k_t^b \leq V_t, \tag{14}$$

where θ is a positive constant and V_t is the franchise value of the bank. The important feature that arises from constraint (14) is that banks' leverage changes with economic conditions. In contrast, constraint (13) implies that banks' leverage is constant. Our preferred interpretation of our setting (13) is that banks have a target level of capital ratio above the regulatory requirement and maintain that level through the cycle.[11,12]

Aggregation

Using $K_{b,t}$, N_t, and D_t to denote the capital stock, net worth, and deposits at the aggregate level, we can express the leverage and balance sheet condition of the banking sector as follows.

$$\begin{aligned} Q_t K_{b,t} &\leq \phi N_t, \text{ and} \\ Q_t K_{b,t} &= N_t + \frac{D_t}{P_t}. \end{aligned} \tag{15}$$

By summing across both surviving and entering banks, we can derive an equation that represents the evolution of bank net worth in the aggregate economy.

$$N_t = \sigma \left[(Z_t + Q_t) K_{b,t-1} - \frac{\overline{R}_{d,t-1}}{\pi_t} \frac{D_{t-1}}{P_{t-1}} \right] + A_t W_b, \tag{16}$$

where $W_b = (1-\sigma) w^b$. Note also that the aggregate consumption of exiting banks is expressed by the following equation.

$$C_{t,b} = (1-\sigma) \left[(Z_t + Q_t) K_{b,t-1} - \frac{\overline{R}_{d,t-1}}{\pi_t} \frac{D_{t-1}}{P_{t-1}} \right]. \tag{17}$$

For the purpose of our analysis, we define NIM as the gap between banks' interest income $\left(\frac{Z_t+Q_t}{Q_{t-1}} - 1\right) Q_{t-1}K_{b,t-1}$ and interest expenses $\left(\frac{\overline{R}_{d,t-1}}{\pi_t} - 1\right) \frac{D_{t-1}}{P_{t-1}}$ divided by interest bearing assets $Q_{t-1}K_{b,t-1}$ following Di Lucido et al. (2017) and Claessens et al. (2018).

[11] There are empirical studies documenting bank behavior that is consistent with our setting. See, for example, Memmel and Raupach (2010), who document the practice of pursuing a target above the level of capital required among German banks.

[12] In the appendix we simulate a model where capital constraint (13) is replaced with incentive constraint (14) and show that the results are qualitatively unchanged.

The lending and deposit spreads, $\tau_{l,t}$ and $\tau_{d,t}$, are defined as follows.

$$
\begin{aligned}
NIM_t &\equiv \frac{\left(\frac{Z_t+Q_t}{Q_{t-1}}-1\right)Q_{t-1}K_{b,t-1}-\left(\frac{\overline{R}_{d,t-1}}{\pi_t}-1\right)\frac{D_{t-1}}{P_{t-1}}}{Q_{t-1}K_{b,t-1}}, && (18)\\
\tau_{l,t} &\equiv \frac{(Z_t+Q_t)}{Q_{t-1}}-\frac{R_{GB,t-1}}{\pi_t}, \text{ and} && (19)\\
\tau_{d,t} &\equiv \frac{R_{GB,t-1}}{\pi_t}-\frac{\overline{R}_{d,t-1}}{\pi_t}. && (20)
\end{aligned}
$$

Following existing studies on the relationship between the level of nominal interest rates and bank profits, we use NIM to measure bank profits. Notice that NIM increases when either spread $\tau_{l,t}$ or $\tau_{d,t}$ widens. This is because by rearranging the terms above, NIM can be expressed as follows.

$$
NIM_t = \tau_{l,t}+\tau_{d,t}+\left(\frac{\overline{R}_{d,t-1}}{\pi_t}-1\right)\frac{1}{\phi}. \tag{21}
$$

3.4 Non-bank lenders

NBLs are financial institutions that finance firms' economic activities without collecting deposits. Unlike banks, they are not subject to capital requirements and their investments $K_{h,t}$ are not affected by their own past profits. Our preferred interpretation is that they include insurance companies, various funds, security firms, and households themselves.

In each period t, NBLs borrow capital stock $K_{h,t}$ from households and collect a management fee $F(K_{h,t})$ that increases with the amount of capital stock $K_{h,t}$ borrowed. They lend the borrowed capital stock to wholesale goods producers, taking prices and interest rates as given. NBLs pay monitoring costs that increase with the size of capital stock that they manage, namely $\alpha A_t K_{h,t}^2$, where $\alpha > 0$ is a parameter that governs the size of the monitoring cost. In the following period NBLs receive earnings Z_t from wholesale goods producers and return earnings $R_{NB,t+1}/P_{t+1}$ and capital stock $K_{h,t}$ to households. Because NBLs are competitive, the following equations hold in equilibrium.

$$F\left(K_{h,t}\right) = \alpha K_{h,t}^{2}, \tag{22}$$

$$\frac{R_{NB,t}}{P_t} = Z_t = (1-\eta)\, A_t \frac{P_{y,t}}{P_t} \left(K_t\right)^{-\eta} \left(L_t\right)^{\eta}. \tag{23}$$

3.5 Government

The government

The government collects lump-sum taxes T_t and seigniorage $(M_t - M_{t-1})/P_t$, and issues new debt B_t/P_t to finance its repayment of outstanding debt $R_{GB,t-1}B_{t-1}/P_t$. Consequently, the following equation holds in each period.

$$\frac{M_{t-1} + R_{GB,t-1}B_{t-1}}{P_t} = T_t + \frac{M_t + B_t}{P_t}.$$

The central bank

The central bank sets the level of the nominal interest rate $R_{GB,t}$ following the Taylor rule described below.

$$R_{GB,t} = \min\left\{1, \overline{R}\overline{\pi}\left(\frac{\pi_t}{\overline{\pi}}\right)^{\varphi_\pi}\right\}, \tag{24}$$

where $\overline{R}$ is the natural rate of interest, $\overline{\pi}$ is the target inflation rate, and $\varphi_\pi > 1$ is the policy weight attached to inflation rate π_t.[13,14,15]

[13] As discussed in Woodford (2003), the effective lower bound of the policy rate $R_{GB,t}$ is a result of the fact that the nominal return from holding money is unity. In the appendix we discuss how the nominal return from holding money M_t matters to the economy.

[14] Obviously, when the recovery rate x_t is equal to or above unity and the probability of a run p_t is zero, the long-run (or equivalently, the steady state) inflation rate in normal times π_t and expected inflation rate $\mathrm{E}_t\left[\pi_{t+1}\right]$ coincide with the target rate of inflation $\overline{\pi}$. When the recovery rate x_t is below unity and a run equilibrium exists, the long-run inflation rate in normal times π_t coincides with the target rate of inflation $\overline{\pi}$ one-for-one, but the expected inflation rate $\mathrm{E}_t\left[\pi_{t+1}\right]$ does not. This is because the inflation rate during a run π_{t+1}^{*} deviates from the target rate $\overline{\pi}$.

[15] More precisely, in the simulation exercises below, we assume that the effective lower bound is 1.0001 so that the return from holding government bonds B_t is always strictly greater than the return from holding money M_t, whose return is unity.

3.6 Market clearing conditions

The market clearing conditions for intermediate goods and wholesale goods are given by

$$\int_0^1 \tilde{y}_t(h)\, dh = y_t.$$

Final goods are consumed by households, invested in claims against NBLs, and used by wholesale goods producers to pay adjustment costs when changing prices. The market clearing condition for final goods is given by

$$C_t + \alpha K_{h,t}^2 + C_{t,b} = Y_t - \frac{\kappa}{2}(\pi_t - \overline{\pi})^2 Y_t. \tag{25}$$

In addition, following Gertler and Kiyotaki (2015), we assume that the total supply of capital stock is fixed so that the following equation holds in every period[16]:

$$K_{h,t} + K_{b,t} = K = 1. \tag{26}$$

3.7 Bank runs and run probability

From Gertler and Kiyotaki (2015), we borrow the condition for a bank run to occur and the assumption about how the probability of a bank run is determined when a bank run equilibrium exists.

Bank runs

Similar to Gertler and Kiyotaki (2015) a bank run occurs when households decide to withdraw their deposits. When a run occurs, banks liquidate their assets $K_{b,t-1}$ by selling them to NBLs, the proceeds of which are collected by households. The recovery rate x_t is then expressed by the following equation.

$$x_t = \frac{(Q_t^* + Z_t^*)\, K_{b,t-1}}{\overline{R}_{d,t-1}\,(\pi_t^*)^{-1}\,(D_{t-1}/P_{t-1})}. \tag{27}$$

[16]This equation also implies that the amount of capital stock K_t used by goods producers, as appears in equation (7), is fixed in equilibrium, namely $K_t = K$.

Q_t^*, Z_t^*, and π_t^* are the price of one unit of capital stock, the MPK, and the inflation rate during a run, respectively. Households have no incentive to withdraw their deposits D_{t-1} so long as the contractual rate $\overline{R}_{d,t-1}$ is expected to be paid. They can choose to run if the numerator of equation (27) is smaller than the denominator, or equivalently, if the recovery rate x_t is smaller than one. In this case, there are two equilibria: one with a bank run and one without a bank run. On the other hand, when $x_t \geq 1$, households are better off keeping their deposits D_t at banks and a bank run does not occur.

As shown below, the recovery rate x_t varies endogenously depending on economic conditions including the level of the nominal interest rate $R_{GB,t}$. A low recovery rate x_t implies that the banking system is more susceptible to a bank run. It is important to note, however, that unless the recovery rate x_t is below unity, a bank run equilibrium does not emerge. In what follows, we use the recovery rate x_t as our measure of stability in the banking sector and examine how the level of the nominal interest rate $R_{GB,t}$ affects this variable.

Bank run probability

When the recovery rate x_t is greater than unity, the probability of a bank run is zero, since households have no incentive to run. When the recovery rate x_t is smaller than unity, a bank run equilibrium emerges and a bank run can take place. The model described so far is, however, silent about how households form their expectations of a run and how they assign a probability to a run state when the recovery rate is below one.

The strategy chosen in Gertler and Kiyotaki (2015) is to simply assume that the following function, which maps the recovery rate x_t to the probability of a run, holds:

$$p_t \equiv\ 1 - \mathrm{E}_t \left[\frac{\left(Q_{t+1}^* + Z_{t+1}^*\right) K_{b,t}}{\overline{R}_{d,t} \left(\pi_{t+1}^*\right)^{-1} \left(D_t/P_t\right)} \right],\ \text{for } \mathrm{E}_t x_{t+1} \leq 1. \tag{28}$$

In contrast to Gertler and Kiyotaki (2015), the focus of our paper is on whether or not the level of the nominal interest rate $R_{GB,t}$ can reduce the recovery rate x_t to a value below unity. Short-run feedback mechanisms from changes in the probability of a bank run p_t

to the macroeconomy are beyond the scope of our paper. For the convenience of analysis, however, we follow Gertler and Kiyotaki (2015) and assume that equation (28) holds in our model, too.

3.8 Calibration

We set the values for conventional parameters following Gertler and Kiyotaki (2015) and other existing studies. The discount factor β is set to 0.995, the labor share in production inputs η is set to 0.66, the utility weight attached to the disutility of labor inputs χ is set to 1, and the inverse of the Frisch elasticity of labor v is also set to 1.

We calibrate four key parameters, ω, μ, α, and σ, so that the steady state value of the endogenous variables, namely the slope of the deposit spread to the nominal interest rate $\Delta\tau_{d,t}/\Delta R_{GB,t}$, the lending spread $\tau_{l,t}$, the ratio of money balances to bank deposits M_t/D_t, and the relative significance of the banking sector $K_{b,t}/K_{h,t}$ in an economy where the annual TFP growth rate z is 1% and the annual target inflation rate $\overline{\pi}$ is 1%, are consistent with the data averaged over the period from 2002 to 2016.[17,18,19] We construct the deposit spread as the difference between the nominal interest rate and the deposit rate, and the lending spread as the difference between the lending rate and the nominal interest rate.[20] We use total deposits for deposits and the currency in circulation for

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[17] We set the annual TFP growth rate to $z = 0.01$ and the annual target inflation rate to $\overline{\pi} = 1.01$ so that they are roughly consistent with the estimated potential growth rate of the Japanese economy released by the Bank of Japan and the long-run inflation expectation or trend inflation rate for Japan estimated by Hogen and Okuma (2018) and Kaihatsu and Nakajima (2018) during the target period.

[18] We choose the target period to begin in 2002 and end in 2016 based on the premise that this period was one in which Japan's economy had been in a state where a bank run equilibrium did not exist, implying that the target inflation rate $\overline{\pi}$ coincided with the expected inflation rate. It is considered that the probability of a bank run, which was above zero during the banking crisis of the 1990s, had fallen significantly by that time due to government initiatives such as capital injections and inspections. Indeed, while the government promised to guarantee bank deposits without limit as an emergency measure in the mid-1990s, it began to implement a deposit insurance cap for time deposits in 2002. Our choice is also consistent with Reinhart and Rogoff (2011), who state that the Japanese banking crisis lasted from 1992 to 2001.

[19] The slope of the deposit spread to the nominal interest rate $\frac{\Delta\tau_{d,t}}{\Delta R_{GB,t}}$ in the model is calculated as the incremental change in the steady state deposit spread $\tau_{d,t}$ when the target inflation rate $\overline{\pi}$ is changed marginally from 1.01.

[20] The nominal interest rate is the one-year government bond yield. The deposit rate is computed by dividing total payments to deposits by total deposits outstanding, using the data of all national banks. Both of these variables are provided by the Japanese Bankers Association. The lending spread is the prime loan rate released by the Bank of Japan.

money holdings. Our measure of the relative size of the banking sector is given by dividing financial assets held by the banking sector by financial assets held by other private financial institutions, as reported in Financial Stability Board (2018).

The values of the model parameters and those of the target data are shown in table 2.

4 Effects of Prolonged Low Nominal Interest Rates

4.1 Effects on bank profits

We study how prolonged low nominal interest rates $R_{GB,t}$ affect bank profits by computing the steady state values of the key bank variables in our model, such as NIM and bank net worth N_t, in an economy with various TFP growth rates z and target rates of inflation $\overline{\pi}$. The reason why we analyze the effects of the two rates, z and $\overline{\pi}$, instead of innovations to the Taylor rule of equation (24) is that we are interested in the effects of prolonged low nominal interest rates $R_{GB,t}$, which are determined by rates z and $\overline{\pi}$. In fact, according to equation (24), in the long-run we have[21]

$$R_{GB,t} = \overline{R}\overline{\pi} = \beta^{-1} e^{z} \overline{\pi}.$$

In computing the steady state values of the variables of interest, we keep all of the parameters, except for the TFP growth rate z or the target inflation rate $\overline{\pi}$, unaltered.

Bank profits with various TFP growth rates z

Figure 5 shows the steady state values of the nominal interest rate $R_{GB,t}$, real deposits D_t/P_t, real money balances M_t/P_t, NIM, the two spreads $\tau_{l,t}$ and $\tau_{d,t}$, gross output Y_t, bank net worth N_t, and the relative size of banks' capital investments compared to NBLs' capital investments $K_{b,t}/K_{h,t}$, in economies with the TFP growth rates z ranging from -2% to +2%. Note that we keep the target inflation rate $\overline{\pi}$ at 1%. As discussed above, with a lower TFP growth rate z, the nominal interest rate $R_{GB,t}$ falls because the natural rate of

[21] Precisely speaking, the natural rate of interest $\overline{R}$ equals $e^{z}\beta^{-1}$ only when the recovery rate x_t exceeds unity so that households attach zero probability to a run state. Even when the recovery rate x_t is below unity, however, the natural rate of interest $\overline{R}$ remains an increasing function of the TFP growth rate z.

interest $\overline{R}$ falls.

Bank profits are small when the nominal interest rate $R_{GB,t}$ is low, as shown by the changes in NIM and bank net worth N_t. Quantitatively, our model predicts an approximately 40 basis point decline in NIM in response to a one percentage point decline in the nominal interest rate $R_{GB,t}$, due to a decline in the TFP growth rate from 1.0% to 0%. This result roughly agrees with estimates from existing empirical studies. For example, Claessens et al. (2018) and Borio et al. (2015) document approximately 20 and 50 basis point declines in NIM, respectively, when the nominal interest rate $R_{GB,t}$ falls by one percentage point from an already low rate.

This decline in bank profits comes from the compression of the deposit spread $\tau_{d,t}$. To see why this happens, we combine equations (3), (5), and (6) in order to derive the following two equations.

$$\beta \mathrm{E}_t\left[\frac{1}{\pi_{t+1}C_{t+1}}\right][R_{GB,t}-1] = \omega\frac{(\frac{M_t}{P_t})^{\zeta-1}}{(\mu\frac{D_t}{P_t})^{\zeta}+(\frac{M_t}{P_t})^{\zeta}} \tag{29}$$

$$\beta \mathrm{E}_t\left[\frac{1}{\pi_{t+1}C_{t+1}}\right][R_{GB,t}-R_{d,t+1}] = \omega\frac{\mu^{\zeta}(\frac{D_t}{P_t})^{\zeta-1}}{(\mu\frac{D_t}{P_t})^{\zeta}+(\frac{M_t}{P_t})^{\zeta}} \tag{30}$$

The first equation shows how households allocate their assets between government bonds B_t and money M_t. The left hand side of the equation represents the spread between holding bonds and holding money, $R_{GB,t}-1$, while the right hand side of the equation represents the marginal utility of holding money M_t. This equation suggests that when the nominal interest rate $R_{GB,t}$ is low, so is the marginal utility of holding money M_t, which implies that money holdings M_t should be large. Notice that because the return from money holdings M_t is always one and does not decline with a decline in the nominal interest rate $R_{GB,t}$, money M_t becomes more attractive, increasing households' demand for M_t. The second equation shows how households allocate their assets between government bonds B_t and deposits D_t. The left hand side of the equation represents the return spread between bonds and deposits, $R_{GB,t}-R_{d,t+1}$, and the right hand side of the equation represents the marginal utility of holding deposits D_t. This equation, combined with equation (20), indicates that when more money M_t is held by households, the deposit spread $\tau_{d,t}$ is compressed. This is because the utility gain from holding deposits D_t, which is captured by the right hand side of equation (30), falls with money M_t, since, as indicated by the positive sign of parameter ζ, bank deposits D_t and money M_t are substitutes. Equations (29) and (30) therefore together imply that a decline in the nominal interest rate $R_{GB,t}$ leads to a decline in the demand for deposits D_t, by increasing the incentive for households to hold more money M_t, and leads to a narrowing of the deposit spread $\tau_{d,t}$.[22,23]

The implication of the narrowing deposit spread $\tau_{d,t}$ for bank profits is straightforward. As the definition of NIM (21) indicates, this narrowing translates into a narrowing of banks' NIM, which then translates into a smaller bank net worth N_t, as less earnings are accumulated into net worth N_t as indicated by equation (16).[24,25] In addition, because of capital requirement constraint (15), a smaller bank net worth N_t reduces the capital stock held by banks $K_{b,t}$, leading to a decline in gross output Y_t. This is because banks outperform NBLs in terms of lending, as indicated by the fact that $\alpha > 0$ in the NBL lending function equation (22). A shift in ownership of capital stock from banks to NBLs causes a loss in resources, as shown in resource constraint (25).

[22] The fact that the deposit spread $\tau_{d,t}$ is compressed by a decline in the TFP growth rate implies that the deposit rate $R_{d,t}$ does not fall one-for-one with a fall in the nominal interest rate $R_{GB,t}$. Indeed, because of the decline in the marginal utility of holding deposits, the deposit rate $R_{d,t}$ falls less. Taking as an example the case of a decline in the TFP growth rate from 2% to 0%, the deposit rate $R_{d,t}$ falls only by 1.3 percentage points while the nominal interest rate $R_{GB,t}$ falls by 2 percentage points.

[23] In figure 5, the deposit spread $\tau_{d,t}$, NIM, and bank net worth N_t monotonically increase with the nominal interest rate $R_{GB,t}$ until the nominal interest rate $R_{GB,t}$ reaches approximately 4%. Beyond that rate, all variables remain almost constant regardless of changes in the nominal interest rate $R_{GB,t}$. This observation stems from the particular property of the utility function that utility gains from liquidity services Ω_t become increasingly satiated as deposits D_t increase.

[24] As equation (21) shows, a decline in the deposit spread $\tau_{d,t}$ translates into a compressed NIM only when the lending spread $\tau_{l,t}$ does not widen much. This is, in fact, the case. When the nominal interest rate $R_{GB,t}$ falls, the lending rate $(Q_t + Z_t)\, Q_{t-1}^{-1}$ does not fall one-for-one, thus widening the lending spread $\tau_{l,t}$. Again, taking the case of a decline in the TFP growth rate from 2% to 0% as an example, the lending rate $(Q_t + Z_t)\, Q_{t-1}^{-1}$ falls only by 1.7 percentage points while the nominal interest rate $R_{GB,t}$ falls by 2 percentage points. One reason for the widening loan spread is that banks have less net worth N_t and face more severe balance sheet constraints. Because they are unable to conduct large scale investments, the lending spread $\tau_{l,t}$ widens. Compared with changes in the deposit spread $\tau_{d,t}$, however, changes in the lending spread $\tau_{l,t}$ are quantitatively small.

[25] The implications of a decline in the TFP growth rate z or of the target inflation rate $\overline{\pi}$ on NIM are little changed even if NIM is alternatively defined as the spread between the lending rate and the deposit rate $(Z_t + Q_t)/Q_{t-1} - \overline{R}_{d,t-1}\pi_t^{-1}$.

Bank profits under various target inflation rates $\overline{\pi}$.

Figure 6 shows the steady state values of the same variables as those shown in figure 5 in economies with various target inflation rates $\overline{\pi}$ ranging from -2% to +2%. Note that in this case the real interest rate $R_{GB,t-1}\pi_t^{-1}$ is fixed at $e^z\beta^{-1}$ and only the target inflation rate $\overline{\pi}$ is changed.

The effects on spreads $\tau_{l,t}$ and $\tau_{d,t}$ and bank profits, as measured by NIM and N_t, when changing the target inflation rate $\overline{\pi}$ are qualitatively similar to those observed when we changed the TFP growth rate z. That is, a low nominal interest rate $R_{GB,t}$ is accompanied by a compressed deposit spread $\tau_{d,t}$ and smaller bank profits. Once again, the key mechanism is households' substitution from bank deposits D_t to money M_t. In the case of a decline in the target inflation rate $\overline{\pi}$, an increase in the return on money holdings π_t^{-1} makes bank deposits less attractive, increasing households' demand for money M_t and compressing the deposit spread $\tau_{d,t}$. NIM and bank net worth N_t fall according to equations (21) and (16), respectively.

4.2 Effect on banking stability

Next, we study how the nominal interest rate $R_{GB,t}$ affects banking stability. As discussed above, we use the recovery rate x_t as our measure of banking stability. When the recovery rate x_t is below unity, a bank run equilibrium emerges and households form their expectations based on equation (28).

Figure 7 shows the steady state value of the recovery rate x_t, the probability of a bank run p_t, and the return on capital stock $(Q_t + Z_t)\, Q_{ss}^{-1}$ during a run in an economy with different TFP growth rates z (left column) and target inflation rates $\overline{\pi}$ (right column). Q_{ss} is the price of capital stock one quarter before a run takes place, which is equivalent to the steady state value of this variable in normal times. The return on capital $(Q_t + Z_t)\, Q_{ss}^{-1}$ is informative for assessing banking stability because it is related to the recovery rate x_t, as shown in the following expression.[26]

[26] In figure 7, because the economy experiences a run in the first quarter, i.e., at $t = 1$, the return on capital $(Q_1 + Z_1)\, Q_{ss}^{-1}$ equals the return on capital $(Q_t^* + Z_t^*)\, Q_{t-1}^{-1}$ shown in equation (31).

$$x_t = \frac{(Q_t^* + Z_t^*)\, K_{b,t-1}}{\overline{R}_{d,t-1}\, (\pi_t^*)^{-1}\, (D_{t-1}/P_{t-1})} = \frac{(Q_t^* + Z_t^*)}{Q_{t-1}} \times \frac{\phi}{\overline{R}_{d,t-1}\, (\pi_t^*)^{-1}\, (\phi - 1)} \tag{31}$$

Panel (a) indicates that banking stability is undermined to an increasing degree as either the TFP growth rate z or the target inflation rate $\overline{\pi}$ falls. For example, with a constant TFP growth rate z, the recovery rate x_t is about 1.08 when the target inflation rate $\overline{\pi}$ is +2% and falls below unity when the target inflation rate $\overline{\pi}$ falls below -1%. As shown in panel (c), such a decline in the recovery rate x_t is caused by a decline in the numerator of equation (27), i.e., the return on capital stock during a run $(Q_t^* + Z_t^*)\, Q_{ss}^{-1}$. The return falls monotonically as either the TFP growth rate z or the target rate of inflation falls. Panel (b) shows how the probability of a bank run p_t changes when the two rates, z and $\overline{\pi}$, change. For both the TFP growth rate z and the target inflation rate $\overline{\pi}$, the probability of a bank run p_t remains zero so long as the two rates, z and $\overline{\pi}$, take positive values, as a bank run equilibrium does not emerge in this region.

Comparison with the model without liquidity services

Why do low nominal interest rates undermine banking stability? Do compressed bank profits play any role? To see if they do, we construct a model that differs from our baseline model only in terms of the household's utility function, which we refer to as the alternative model in this subsection. In the alternative model, we assume that the household's utility function is given as follows, rather than the function given in equation (1).

$$U_t \equiv \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \left[\log(C_{t+i}) - \chi \frac{L_{t+i}^{1+v}}{1+v} - \tilde{\mathrm{I}}_{t+i} \delta \log\left(\frac{D_{t+i-1}}{P_{t+i-1}} \right) \right]. \tag{32}$$

In this case, the household does not receive utility from holding bank deposits D_t/P_t or money M_t/P_t, as is the case in Gertler and Kiyotaki (2015). Consequently, the deposit spread $\tau_{d,t}$ does not reflect these gains, taking a value of zero, so far as the recovery rate x_t is above unity. The other settings of the alternative model are the same as those in the baseline model.[27] As shown below, the implications of prolonged low nominal interest

[27] In the alternative model, the household does not have a reason to hold money M_t because utility

rates $R_{GB,t}$ in the alternative model are starkly different from those in the baseline model since bank profits are no longer affected by nominal interest rates $R_{GB,t}$. Comparison of the two models therefore allows one to isolate the effects of nominal interest rates $R_{GB,t}$ on banking stability that arise from changes in the deposit spread $\tau_{d,t}$ from those that arise from other channels.

Figure 8 shows how the key variables, the policy rate $R_{GB,t}$, the inflation rate π_t, the return on capital $(Q_t + Z_t)\,Q_{ss}^{-1}$, and bank deposits D_t/P_t, evolve after a run, as the target inflation rate $\overline{\pi}$ ranges from -1% to +5%, under the two models. It can be observed that in both models a run exerts downward pressure on both the inflation rate π_t and the return on capital $(Q_t + Z_t)\,Q_{ss}^{-1}$, and temporarily reduces bank deposits D_t/P_t.[28]

Two observations are notable. The first is that the ELB of the policy rate $R_{GB,t}$ plays a role in determining the recovery rate x_t. This can be readily seen in the simulation results based on the alternative model. Because a run imposes deflationary pressure on the economy, the central bank cuts the nominal interest rate $R_{GB,t}$. When the target inflation rate is above -1%, the nominal interest rate $R_{GB,t}$ and the inflation rate π_t move in parallel after a run and the time path of the return on capital $(Q_t + Z_t)\,Q_{ss}^{-1}$ becomes the same for all levels of the target inflation rate. In contrast, when the target inflation rate $\overline{\pi}$ is -1%, the nominal interest rate $R_{GB,t}$ hits the ELB during a run. The time path of the inflation rate π_t no longer moves in parallel with the nominal interest rate $R_{GB,t}$ and the return on capital $(Q_t + Z_t)\,Q_{ss}^{-1}$ falls more than in other cases.

The second observation is that the ELB is only part of the story and that utility from liquidity services also plays a role. This can be seen clearly in the simulation results based on the baseline model. In the baseline model, developments in the return on capital

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gains from liquidity services are absent. Consequently, the budget constraints of the household and the government are also different from the baseline model in the sense that money M_t is absent from the equations.

[28] The nature of a run in our models is equivalent to that in Gertler and Kiyotaki (2015). During a run, households withdraw deposits D_t from banks, banks' capital investment $K_{b,t}$ falls to zero, and NBLs' capital investment $K_{h,t}$ becomes one due to the fixed supply assumption on the capital stock K (26). A run lasts one quarter but its effects last longer. As shown in figure 8, banks gradually accumulate net worth N_t and start lending to firms after a run has ended. Similar to Gertler and Kiyotaki (2015), economic conditions, such as the depth of output decline, during a run are independent from the state variables in the economy, including the size of banks' capital holdings $K_{b,t}$, in the period prior to the run.

$(Q_t + Z_t)\, Q_{ss}^{-1}$ and the pace at which deposits D_t/P_t recover after a run differ when the target inflation rate $\overline{\pi}$ is 3% or 5%, even though the ELB does not bind in either case. This is in contrast to the alternative model, where both the return on capital $(Q_t + Z_t)\, Q_{ss}^{-1}$ and deposits D_t/P_t follow identical time paths, regardless of whether the target inflation rate $\overline{\pi}$ is 3% or 5%.

Similar observations can be made from studying the results of the steady state analysis. Figure 9 shows how the steady state values of the key variables in the two models change with the TFP growth rate z and the target inflation rate $\overline{\pi}$. Variables computed based on the alternative model are indicated by the solid lines and those computed based on the baseline model are indicated by the dotted lines. The vertical lines in each panel represent the highest TFP growth rate z or the highest target inflation rate $\overline{\pi}$ at which the ELB binds after a run. Again, two points are notable. Firstly, under the alternative model, so long as the ELB does not bind, the recovery rate x_t remains constant regardless of the value of the TFP growth rate z or the target inflation rate $\overline{\pi}$.[29] In contrast, in the regions where the ELB binds, the recovery rate x_t falls with a decline in the nominal interest rate $R_{GB,t}$. Secondly, under the baseline model, the recovery rate x_t increases monotonically with the two rates, z and $\overline{\pi}$, even in the region where the ELB does not bind suggesting that changes in utility from liquidity services affect the recovery rate x_t.

One other takeaway from figure 9 is that the recovery rate x_t does not fall one-for-one with a decline in bank net worth N_t. For example, a comparison of the two models shows that while bank net worth N_t in the baseline model is always higher than that in the alternative model, the recovery rate x_t in the former is lower than that in the latter for a wide range of values for the two rates, z and $\overline{\pi}$. In addition, simulation results based on the baseline model show that the marginal widening of the deposit spread $\tau_{d,t}$ falls as the nominal interest rate $R_{GB,t}$ rises. This is in contrast to the recovery rate x_t, which

[29] Note that because utility from liquidity service is absent in the alternative model, the deposit spread $\tau_{d,t}$ is zero in the region where a run equilibrium does not exist, regardless of the values of the TFP growth rate z and the target inflation rate $\overline{\pi}$. Consequently, NIM and bank net worth N_t also become independent from the two rates, z and $\overline{\pi}$. In the region where the recovery rate x_t is below one, bank deposits D_t are no longer safe assets. The deposit spread $\tau_{d,t}$ therefore deviates from zero, affecting all bank-related variables, such as bank net worth N_t and spreads $\tau_{l,t}$ and $\tau_{d,t}$. The quantitative effects that arise from changes in the deposit spread $\tau_{d,t}$ are, however, minor.

continues to rise with a rise in the nominal interest rate $R_{GB,t}$.

The two channels from which nominal interest rates affect banking stability

Next, we return to the definition of the recovery rate x_t. The recovery rate x_t can be expressed as the difference of two variables, the MPK Z_t and the capital stock held by banks $K_{b,t}$, in normal times and in periods with a bank run. From equation (4), we have

$$Q_{ss} = \sum_{i=0}^{\infty} \beta^i \left(\beta Z_{ss} + 2\alpha \left(K_{b,ss} - 1 \right) \right). \tag{33}$$

This equation shows that the price of capital Q_{ss} in normal times increases with both the MPK Z_{ss} and banks' capital investment $K_{b,ss}$.[30] From the same equation (4), we also have

$$\frac{(Q_t^* + Z_t^*)}{Q_{ss}} = \sum_{i=0}^{\infty} \beta^i \frac{C_t}{C_{t+i}} \frac{Z_{t+i}}{Q_{ss}} + 2\alpha \sum_{i=0}^{\infty} \beta^i \frac{C_t}{C_{t+i}} \frac{(K_{b,t+i} - 1)}{Q_{ss}}. \tag{34}$$

Equations (33) and (34) indicate that the recovery rate x_t falls when the MPK Z_t, banks' capital stock holding $K_{b,t}$, or both fall to less than their pre-crisis levels, Z_{ss} and $K_{b,ss}$, following a run.

How are the two channels described so far related to equations (33) and (34)? Firstly, when the ELB binds, the adverse effects of a run are mitigated to a smaller degree by the central bank, leading to a greater decline in the MPK Z_t^* and the inflation rate π_t^*. A low MPK Z_t^* during a run reduces the recovery rate x_t, as indicated by equation (31), and a lower inflation rate π_t^* during a run slows down the recovery of bank net worth N_t, resulting in a smaller banking sector $K_{b,t}$.

Secondly, even if the ELB does not bind, the nominal interest rate $R_{GB,t}$ falls to an even lower rate during a run if the steady state nominal interest rate $R_{GB,t}$ is low. Because NIM increases with the nominal interest rate $R_{GB,t}$, a higher nominal interest rate $R_{GB,t}$ in normal times increases the pace at which net worth N_t grows, leading to a greater accumulation of bank capital $K_{b,t}$ in the period following a run.

[30] The variables Z_{ss} and $K_{b,ss}$ stand for the steady state values of the MPK Z_t and banks' capital investment $K_{b,t}$, respectively, when the recovery rate x_t is greater than unity.

Figure 10 shows the time path of the eight key variables after a run under the baseline model, with six different target inflation rates $\overline{\pi}$. Developments of the variables are consistent with the explanations provided for the two channels above. Firstly, the adverse effects of a run on the inflation rate π_t and gross output Y_t are larger, leading to a larger decline in the MPK Z_t, the return on capital $(Q_t + Z_t)\, Q_{ss}^{-1}$, and the capital stock held by banks $K_{b,t}$, when the nominal interest rate $R_{GB,t}$ hits the ELB. In addition, when comparing the two cases where the ELB does not bind, i.e., cases where the target inflation rate $\overline{\pi}$ is 3% or 5%, the adverse effects on bank net worth N_t and bank capital $K_{b,t}$ are smaller when the target inflation rate $\overline{\pi}$ is 5%.

Equations (33) and (34) also indicate why bank net worth N_t is not closely linked to the degree of banking stability, given by the recovery rate x_t, as shown in figure 9. On the one hand, a large bank net worth N_t in normal times implies that banks can recover their net worth N_t quickly following a run. On the other hand, with a large net worth, the price of capital Q_{ss} in normal times is higher since banks' demand for capital stock $K_{b,ss}$ increases with bank net worth N_t, thus reducing the recovery rate x_t. The net effect is therefore given by how the underlying determinants of bank net worth N_t in normal times help banks recover their net worth N_t after a run. For example, as shown in figures 8 and 9, in an economy where the nominal interest rate $R_{GB,t}$ is sufficiently high, as is the case when the target inflation rate $\overline{\pi}$ is above +1%, the effects of a marginal rise in the nominal interest rate in normal times $R_{GB,t}$ on the recovery pace of bank net worth N_t at a run are noticeably large, even though the effect on bank net worth N_t in normal times is small.

5 International Comparisons

In the previous section, we derived three implications regarding how prolonged low nominal interest rates $R_{GB,t}$ affect banks. Firstly, they compress the deposit spread $\tau_{d,t}$ and NIM and reduce bank net worth and size N_t. Secondly, they undermine banking stability, bringing the economy to a state where a bank run equilibrium exists. In addition, based on our calibration, their quantitative effects on banking stability are not necessarily substantial in the sense that the recovery rate x_t does not fall below unity except for in severe scenarios where the TFP growth rate z or the target inflation rate $\overline{\pi}$ falls below zero.

In this section, we calibrate our baseline model to other developed countries. We do this for two reasons. Firstly, we would like to investigate the extent to which the three findings above can be generalized. Secondly, we would like to uncover the key attributes of banks' business environments that are responsible for these findings.

5.1 Simulation results

Calibration

We calibrate the four key parameters, the monitoring costs of NBL lending α, the liquidity of bank deposits compared to money μ, the utility weight attached to liquidity services ω, and the survival rate of banks σ, using the actual data of the following four variables: the slope of the deposit spread to the nominal interest rate $\Delta\tau_{d,t}/\Delta R_{GB,t}$, the lending spread $\tau_{l,t}$, money balance over bank deposits M_t/D_t, and banks' share of financial assets $K_{b,t}/K_{h,t}$.[31] Similarly to the case of calibration for Japan, we choose the parameter values so that the data coincide with the steady state values of these four endogenous variables in an economy where the TFP growth rate z is 2% and the target inflation rate $\overline{\pi}$ is also 2%.[32] We also choose banks' endowment w_b so that the ratio of w_b to bank net worth N_t at the steady state is 0.05. Other model parameter values are set to the same values as those used for the simulation for Japan.

Table 3 shows the list of calibrated parameter values for Canada, the U.K., and the U.S., together with those of Japan at the top, and the four target variables in the data at the bottom. The size of the NBL monitoring parameter α and the utility weight attached

[31] An alternative way of estimating $K_{b,t}/K_{h,t}$ would be to use the ratio of bank credit to total private sector funding, as used in Gambacorta et al. (2014). Based on the results reported in Gambacorta et al. (2014), the estimate of $K_{b,t}/K_{h,t}$ is the lowest for the U.S., at approximately 0.3, and high for the other three countries, at approximately 0.7.

[32] We choose the values of the TFP growth rate z and the target inflation rate $\overline{\pi}$ so that these are generally consistent with the actual growth rate and the inflation rate from 2002 to 2016 in the three countries, implicitly assuming that the economy was in a state without a bank run equilibrium during this period. We choose this sample period because the data for $K_{b,t}$ and $K_{h,t}$ are available only from 2002 to 2016. For Canada, this treatment is consistent with Reinhart and Rogoff (2011) who report no banking crisis during the period. In contrast, Reinhart and Rogoff (2011) report that there were banking crises in the period from 2007 to 2009 in the U.K. and from 2007 to 2010 in the U.S. Model parameters are little changed, however, even when we drop the years when banking crises took place from the sample for the calibration.

to liquidity services ω are low in Canada and the U.S. and high in Japan and the U.K. Cross-country differences are less pronounced for μ and σ.[33]

Simulations

Figure 11 shows the steady state values of NIM, the two spreads, $\tau_{d,t}$ and $\tau_{l,t}$, and the recovery rate x_t, as a function of the TFP growth rate z for the three countries, keeping the target inflation rate $\overline{\pi}$ at 2%. All of the panels agree with our three earlier findings. Bank profits are compressed and the recovery rate x_t approaches unity, as the TFP growth rate z declines. A decline in the TFP growth rate z does not bring the economy to a state where a run equilibrium exists unless the TFP growth rate z is extremely low.

It is notable, however, that there is large cross-country heterogeneity regarding the level of the recovery rate x_t and how the recovery rate x_t reacts to a change in the TFP growth rate z. For example, when the TFP growth rate z is 0%, the recovery rate x_t in Canada and the U.S. is 1.12 and 1.11, while the value for Japan and the U.K. is 1.04 and 1.03. When the TFP growth rate z falls from +2% to -2%, the recovery rate x_t in the former two countries falls by 0.01 points, whereas that in the latter two countries falls by 0.09 and 0.02 points.[34]

Figure 12 conducts a similar exercise, changing the target inflation rate $\overline{\pi}$, while keeping the TFP growth rate z at 0.02. The results are qualitatively unchanged.

5.2 The Effects of Changes in Technology Parameters

Why do we see differences across countries in the way that banking stability reacts to changes in the nominal interest rate $R_{GB,t}$? What is the key driver of the transmission of the nominal interest rate $R_{GB,t}$? To answer these questions, we focus on three parameters,

[33] It is also notable that the size of banks' endowment w_b differs across countries, reflecting the heterogeneity in the size of bank net worth N_t.

[34] The level of the nominal interest rate $R_{GB,t}$ for Japan is not comparable with that of other countries in figure 11. For example, when the TFP growth rate z is zero in the figure, the nominal interest rate $R_{GB,t}$ is $1.01\beta^{-1}$ for Japan while it is $1.02\beta^{-1}$ for the other three countries due to differences in the target inflation rate $\overline{\pi}$.

α, μ, and ω, and study how the model results change with the values of these parameters.[35]

Productivity of NBLs

The productivity of NBLs is captured by the size of the monitoring cost α. As equation (4) and (22) show, a lower value of α implies that NBLs are more efficient, which in turn implies that NBLs hold a larger share of the capital stock in the economy.

Figure 13 shows the effects of a change in this parameter on bank profits and banking stability.[36] The four top panels show NIM, the two spreads, $\tau_{d,t}$ and $\tau_{l,t}$, the relative size of capital stock held by banks $K_{b,t}/K_{h,t}$, the return on capital during a run $(Q_t^* + Z_t^*)\, Q_{ss}^{-1}$, and the recovery rate x_t as a function of parameter α. For the purposes of exposition, we normalize the value of this parameter by the value for Japan.

When the parameter α takes a small value, NIM is compressed through changes in the lending spread $\tau_{l,t}$, reducing the size of the banking sector $K_{b,t}/K_{h,t}$. In contrast, the return on capital during a run $(Q_t^* + Z_t^*)\, Q_{ss}^{-1}$ and the recovery rate x_t rise. In other words, efficient NBLs reduce bank profits but enhance banking stability. There are two opposing forces. On the one hand, as shown in equation (33), which determines Q_{ss}, a lower monitoring cost for NBLs pushes up the price of capital stock, which in turn leads to a narrower lending spread $\tau_{l,t}$ and NIM. Consequently, bank net worth N_t and bank capital investment $K_{b,t}$ fall. Compressed NIM hampers the accumulation of bank net worth after a run, reducing the recovery rate x_t. On the other hand, as shown in equation (34), a lower monitoring cost for NBLs increases the return on capital after a run $(Q_t^* + Z_t^*)$. When a run occurs, efficient NBLs purchase liquidated bank assets $K_{b,t-1}$ at a higher price, since they can make better use of the capital stock $K_{b,t-1}$. Consequently, the fall in the liquidation price of bank assets $K_{b,t-1}$ is mitigated. As figure 13 shows, the latter effect

[35] To save space, we do not show results regarding how changes in banks' survival probability parameter σ in equation (16) affect bank profits and banking stability. This parameter governs the accumulation of bank net worth N_t, and, given NIM, the aggregate bank net worth N_t grows quickly when σ is large. Based on our simulation, a larger σ decreases bank profits by compressing the lending spread $\tau_{l,t}$ and reduces the recovery rate x_t. The key mechanism is the increase in the price of capital Q_t due to an increase in bank net worth N_t. Quantitatively, however, the cross-country differences in parameter σ are small and the effects on bank profits and banking stability of changing the parameter are minor, particularly, relative to those of parameter α.

[36] In the simulations we maintain other parameter values unchanged.

dominates quantitatively.

It is also worth noting that the value of parameter α varies substantially across countries and plays a quantitatively important role in determining the recovery rate x_t. The parameter value of $\alpha/\alpha_{baseline}$ ranges from 0.6 in Canada to 1.1 in the U.K. Based on panels (c) and (d), an increase in $\alpha/\alpha_{baseline}$ from 0.6 to 1.1 reduces the recovery rate x_t by 0.1 points, which is roughly comparable to the difference in the recovery rate x_t between the U.S. (or Canada) and Japan (or the U.K.) in economies with a target inflation rate $\overline{\pi}$ of 0%.

The bottom two panels show the boundary of the two states, with and without a bank run equilibrium, as a function of NBLs' monitoring cost parameter α and the TFP growth rate z (left panel) or the target inflation rate $\overline{\pi}$ (right panel). The degree of NBL efficiency alters the effects of nominal interest rates $R_{GB,t}$ on banking stability in an important manner, namely by making the economy more resilient to the adverse effects that arise from low nominal interest rates $R_{GB,t}$. For example, in an economy where the monitoring cost parameter α is about the same as that in Japan, all else equal, a bank run equilibrium emerges when the target inflation rate $\overline{\pi}$ falls below -1%. In contrast, with a smaller value of $\alpha/\alpha_{baseline}$, say 0.8, a bank run equilibrium does not emerge unless the target inflation rate $\overline{\pi}$ falls below -1.5%.

Liquidity of deposits

The utility weight μ attached to deposits D_t in equation (2) represents the degree of liquidity services provided by deposits D_t relative to money M_t. Clearly, the parameter value increases if households prefer holding deposits rather than money.

Figure 14 shows the effects of changes in the parameter μ on bank profits and banking stability. A larger μ increases NIM through a widening of the deposit spreads $\tau_{d,t}$ and modestly expands the scale of the banking sector $K_{b,t}$. One can consider this as reflecting an increase in households' demand for bank deposits D_t, as indicated in equation (30). The quantitative effect on banking stability is, however, minor. For example, the recovery rate x_t is altered by 0.016 points when the relative size of the parameter $\mu/\mu_{baseline}$ varies from

0.5 to 1.0., which contrasts with parameter α, which changes the recovery rate x_t by 0.1 points when its relative size $\alpha/\alpha_{baseline}$ varies from 0.5 to 1.0.

The bottom two panels show the boundary of the two states as a function of the liquidity of deposits μ and the TFP growth rate z (left panel) or the target inflation rate $\overline{\pi}$ (right panel). The degree of liquidity of deposits modestly changes how the nominal interest rate $R_{GB,t}$ affects banking stability. For example, when μ is large, a bank run equilibrium is more likely to emerge for the same target inflation rate $\overline{\pi}$. It is notable, however, that its quantitative effect is small compared with that of parameter α.

Utility gains from liquidity services

The utility weight ω attached to liquidity services in equation (1) represents the size of the utility gain that households receive from the liquidity services provided by deposits D_t and money M_t. Figure 15 shows the effects of a change in this parameter ω on bank profits and banking stability. Similar to the effects of a change in parameter μ, a higher ω increases bank profits, through a widening of the deposit spread $\tau_{d,t}$. On the other hand, it reduces the recovery rate x_t since, as shown in panel (d), larger profits expand the size of banks and push up the price of capital Q_{ss}.

The quantitative importance of parameter ω is smaller than that of the monitoring costs of NBLs α and larger than that of the liquidity of deposits μ. For example, the recovery rate x_t is altered only by 0.024 points when the relative size of the parameter $\omega/\omega_{baseline}$ varies from 0.5 to 1.0., while the recovery rate x_t is altered by 0.1 points and 0.016 points, respectively, when the relative sizes of parameters $\alpha/\alpha_{baseline}$ and $\mu/\mu_{baseline}$ vary by the same amount.

The bottom two panels show the boundary of the two states as a function of utility gains from liquidity services ω and the TFP growth rate (left panel) or the target inflation rate $\overline{\pi}$ (right panel). The size of parameter ω barely changes how the nominal interest rate $R_{GB,t}$ affects banking stability. As shown in the bottom panels, the division line is almost independent of the value of parameter ω.

6 Conclusion

Do prolonged low nominal interest rates reduce bank profits and undermine banking stability? To answer to these questions, we extend the model of Gertler and Kiyotaki (2015) and construct a dynamic stochastic general equilibrium model that is specifically designed to address the role of liquidity services provided by bank deposits and the role of competition between banks and other financial institutions. We then theoretically explore how changes in the long-run nominal interest rate affect bank profits and banking stability in the long-run, using the model calibrated to the Japanese economy.

We show that, regardless of whether it is caused by a low natural rate of interest or a low target inflation rate, a prolonged low nominal interest rate compresses the deposit spread by creating an incentive for households to hold money instead of deposits. In addition, it reduces bank profits. We also show that prolonged low nominal interest rates undermine banking stability by reducing the liquidation price of banks assets during a run and bringing the economy closer to a state where a bank run equilibrium exists. Quantitatively, however, their effects on banking stability are not necessarily substantial. Based on our benchmark calibration, low nominal interest rates do not bring the economy to an equilibrium with a bank run unless the TFP growth rate or the target inflation rate takes a negative value. In particular, their effects are limited in an economy where non-bank lenders play a dominant role in lending.

There are three caveats regarding our analysis. Firstly, there are some features of banks that are not addressed but could potentially affect how nominal interest rates affect banks. These include, for example, the term premium, banks' relationships with their wholesale depositors, and banks' market power.[37] Secondly, we consider only conventional monetary policy, and do not study the effects of unconventional policy tools, such as negative interest rate policy and asset purchases. Theoretically, however, if households believe that these tools will be implemented during a run and the expected liquidation value of banks' assets consequently rises, then the recovery rate may rise to a value higher than considered in the

[37] See English (2002), Gambacorta (2014), and Drechsler et al. (2017) for a related discussion.

current paper. Thirdly, while we assume an orderly liquidation of bank assets during a run, in actual bank runs market imperfections may lead to cascades in asset prices, dragging down these prices even further, as emphasized in studies on fire-sales, such as Shleifer and Vishny (2011). Extending the current model in these directions is left for future research.

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A Sensitivity Analysis

In this section, we conduct three separate analyses.

- Analysis of a model where bank leverage changes with economic conditions.
- Analysis of a model where the size of disutility of having deposits δ is absent or is calibrated to values other than that used in the baseline model.
- Analysis of a model where interest is paid on money holdings.
- Analysis of a model where banks are more broadly defined.

A.1 Endogenous Leverage

Based on empirical evidence including Memmel and Raupach (2010), we assume that banks target a specific capital ratio above the regulatory requirement and fix the ratio at a constant level in the baseline model. Practically speaking, however, banks are free to choose their leverage ϕ as long as the ratio does not violate the regulatory requirement. We, therefore, relax this assumption, and study the implications of a model where leverage constraint (13) is replaced with the incentive constraint described in equation (14). The rest of the model settings are unchanged from the baseline model. We refer this model as the GK model.[38]

When the incentive constraint is in place, banks are able to choose the size of their leverage optimally, depending on economic conditions. All else equal, when the continuation value of banks V_t rises due to, for example, the widening of NIMs, banks have an added incentive to expand their balance sheets. Otherwise, banks reduce their balance sheets. As equation (31) implies, the recovery rate x_t becomes lower when bank leverage ϕ_t is higher.

[38] As discussed in Section 4, the model in Gertler and Kiyotaki (2015) abstracts from utility gains from liquidity services Ω_t and, therefore, does not address the channels through which changes in the level of nominal interest rates $R_{GB,t}$ affect the deposit spread $\tau_{d,t}$ and bank profits. In this sense, our GK model differs from the model in Gertler and Kiyotaki (2015).

Appendix Figure 1 shows the steady state values of the key variables computed under various target inflation rates $\overline{\pi}$ in the GK model. All of the transmission mechanisms of prolonged low nominal interest rates on bank profits and banking stability that operate in the baseline model are present. This can be seen in panel (b) which indicates a positive relationship between measures of bank profits and the target inflation rate $\overline{\pi}$ and in panel (e) which indicates a kink in the recovery rate x_t when the target inflation rate $\overline{\pi}$ falls to -1%. In addition to these mechanisms, in the GK model, endogenous changes in bank leverage $Q_tK_{b,t}/N_t$ play an important role. As indicated by panels (b) and (d), bank leverage $Q_tK_{b,t}/N_t$ decreases as NIM falls, because banks are better off with a smaller balance sheet when NIM is compressed.[39] All else equal, a lower bank leverage reduces an incentive for households to run because it indicates that bank debt D_t is small compared with the size of bank assets $Q_tK_{b,t}$.

It is also notable that allowing endogenous changes in bank leverage ϕ_t does not alter our qualitative implications. Though banks choose a lower leverage under prolonged low nominal interest rates, which improves banking stability, other forces that undermine banking stability continue to operate. Indeed, as shown in panel (f), the recovery rate x_t falls.

A.2 Disutility of Deposits during a Run

Early studies on the effects of low nominal interest rates advocated that low nominal interest rates may change the risk taking behavior of banks, adding another class of risks to financial stability. Along these lines, some empirical studies, such as Maddaloni and Peydro (2011), document that banks' lending standards loosen when policy rates are low.

In our model, the implications of differences in risk taking behavior of banks can be addressed by simulating the model with various values of parameter δ. This parameter affects households' disutility of having deposits D_t in the beginning of a run state. With a smaller value for δ households experience less disutility from holding deposits D_{t-1}. Banks

[39] This decreasing bank leverage, in response to a decline in nominal interest rates, is, in fact, consistent with the argument that low nominal interest rates reduce banks' risk taking.

are therefore able to collect deposits at a lower cost, i.e. a lower deposit rate, when the nominal interest rate $R_{GB,t}$ is sufficiently low so that the probability of a bank run p is positive.

Appendix Figure 2 shows the steady state value of the key variables for $\delta = 0$, 2, 3, and 5 as a function of the target inflation rate $\overline{\pi}$. It can be seen that a lower disutility of holding deposits during a run exerts an expansionary effect on production while undermining banking stability. That is, gross output Y_t rises but the recovery rate x_t falls. These observations are consistent with the view that stresses the risk taking channel. The gross output Y_t rises because with larger profits the banking sector becomes larger and fewer resources are spent on monitoring activities. Regarding the recovery rate x_t, there are two forces operating in opposite directions. Firstly, bank profits rise due to lower deposit rates, which increases the pace of bank recovery after a run, pushing up the numerator of the expression (34). Secondly, larger bank profits lead to a larger banking sector in normal times, reducing the denominator of the expression (34). As panel (f) shows, the latter force dominates.

It is notable that value of the parameter δ affects the probability of a bank run p_t only for the case when $p_t > 0$. In other words, the value does not affect the rate of the nominal interest rate $R_{GB,t}$ below which a run equilibrium can emerge. As shown in equation (1), this is because the parameter δ takes a nonzero value and affects the equilibrium conditions only in a state where the recovery rate x_t is below unity.

A.3 Interest Rates Paid on Money

As discussed in Section 4, the key reason why low nominal interest rates $R_{GB,t}$ affect bank profits and banking stability is that households demand money M_t rather than deposits D_t when rates are low. The shift in the demand occurs because the nominal return from holding money is one, regardless of the level of the nominal interest rate $R_{GB,t}$, and a decline in the nominal interest rate $R_{GB,t}$ compresses the spread between the interest rate paid on government bonds B_t and that paid on money M_t.

In order to see this from a different angle, we follow Woodford (2003) and study a hypothetical case in which the government pays interest on money, which we denote as $R_{m,t}$, so that the spread between the interest rate on government bonds B_t and money M_t is constant regardless of the level of the nominal interest rate $R_{GB,t}$. Because households receive interest payments from holding money, Eular equation (6) is replaced with the following expression.

$$\begin{aligned}\frac{1}{C_t} &= \beta \mathrm{E}_t\left[\frac{R_{m,t}}{\pi_{t+1}C_{t+1}}\right] + \omega\frac{(\frac{M_t}{P_t})^{\zeta-1}}{(\mu\frac{D_t}{P_t})^{\zeta}+(\frac{M_t}{P_t})^{\zeta}}. \\ &= \beta \mathrm{E}_t\left[\frac{\psi R_{GB,t}}{\pi_{t+1}C_{t+1}}\right] + \omega\frac{(\frac{M_t}{P_t})^{\zeta-1}}{(\mu\frac{D_t}{P_t})^{\zeta}+(\frac{M_t}{P_t})^{\zeta}},\end{aligned} \tag{35}$$

where $\psi \in (0,1)$ is a parameter that governs the two interest rates, $R_{m,t}$ and $R_{GB,t}$. Note that in this economy the nominal interest rate $R_{GB,t}$ does not have a floor below which it cannot fall. By assumption, the interest rate paid on money $R_{m,t}$ is always smaller than the nominal interest rate $R_{GB,t}$ and therefore there is no rationale for a lower bound on the interest rate.[40]

Appendix Figure 3 shows the steady state values of the key variables in this economy for different values of the target inflation rate $\overline{\pi}$.[41] It is clear that changes in the nominal interest rate $R_{GB,t}$ do not affect bank profit measures, such as the spread $\tau_{d,t}$ and net worth N_t, or the recovery rate x_t, our baking stability measure. To understand the reason for this, it is useful to rearrange the equations (3), (5), and (35) to derive the following expression.

$$\frac{R_{GB,t}}{R_{m,t}} = \psi = 1 + \omega\frac{(\frac{M_t}{P_t})^{\zeta-1}}{(\mu\frac{D_t}{P_t})^{\zeta}+(\frac{M_t}{P_t})^{\zeta}} \times C_t \tag{36}$$

$$\frac{R_{GB,t}}{R_{d,t}} = 1 + \omega\frac{\mu^{\zeta}(\frac{D_t}{P_t})^{\zeta-1}}{(\mu\frac{D_t}{P_t})^{\zeta}+(\frac{M_t}{P_t})^{\zeta}} \times C_t \tag{37}$$

[40] See the discussion in Section 4.2 of Woodford (2003). See also Eggertsson et al. (2017) where holding money is assumed to be costly so that the lower bound of the nominal interest rate is affected.

[41] For simplicity, we focus on cases where the recovery rate x_t is above unity.

Similar to the discussion in Woodford (2003), equation (36) says that it is the spread between the rate of return on government bonds B_t and the rate of return on money M_t that matters to money holdings M_t. When this spread $R_{GB,t}R_{m,t}^{-1}$ is fixed at ψ, the demand for money M_t is unaffected by changes in the level of the nominal interest rate $R_{GB,t}$. When the size of money holdings M_t is unaltered, equation (37) shows that the deposit spread $R_{GB,t}R_{d,t}^{-1}$ is also unaltered, leaving the rest of the economy independent from changes in the nominal interest rate $R_{GB,t}$.

A.4 Banks that are more broadly defined

Because of our focus on banking stability, we have so far assumed that NBLs are not subject to runs. However, as seen, for example in the U.S. experience during the global financial crisis, there are financial institutions outside the banking sector that are involved in transactions that can lead to runs. In this subsection, we study the implications of prolonged low nominal interest rates when NBLs are also subject to runs.

We address this issue by simulating the model in which banks are more broadly defined. In the baseline model, we choose the parameter values so that the model-generated steady state values agree with the actual data for the four variables, the slope of the deposit spread to the nominal interest rate $\Delta\tau_{d,t}/\Delta R_{GB,t}$, bank deposits divided by money holdings D_t/M_t, and banks' share of financial assets $K_{b,t}/K_{h,t}$. To construct the data for the fourth variable, we divide financial assets held by banks divided by those held by other financial institutions. Instead, in the alternative model, we construct the data for the fourth variable by dividing financial assets held by banks plus shadow banks by financial assets held by the rest of financial institutions.[42] With this adjustment, the value of banks' share of financial assets $K_{b,t}$ for the U.S. at the steady state rises from 0.28 to 0.49.[43] We then calibrate

[42] We borrow the data from FSB (2018) regarding financial assets held by shadow banks. FSB (2018) focuses on the functions of financial institution and report the size of financial assets held by non-bank entities that may be engaged in credit intermediation that involves liquidity/maturity transformation and/or leverage as that of shadow banks. These entities include a broad range of non-bank financial institutions, such as funds, broker-dealers, securities finance companies, credit insurance companies, and securitization vehicles.

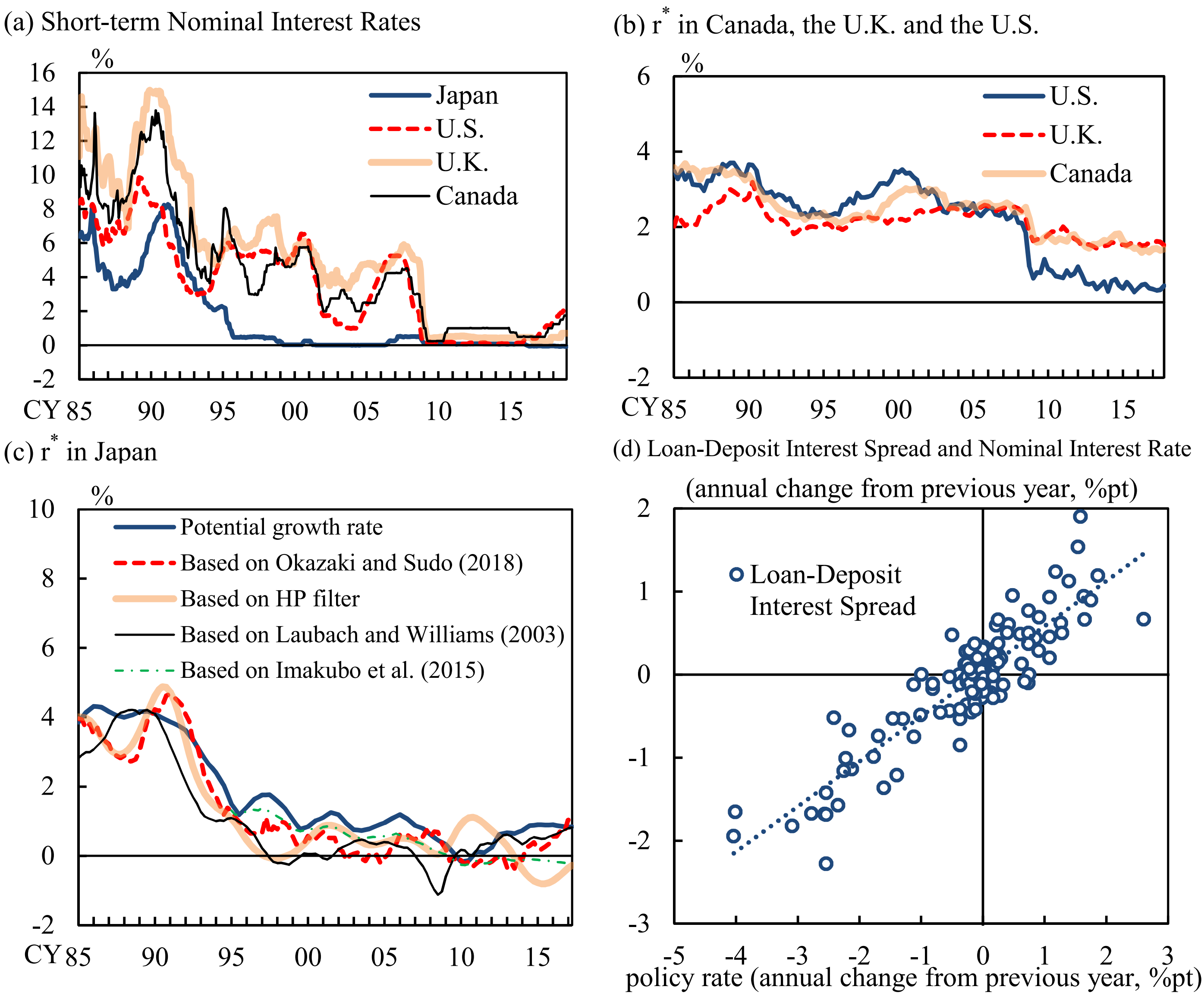
[43] We use the average from 2002 to 2016. For Canada, the U.K., and Japan, the value for K_b becomes 0.43, 0.59, and 0.73, respectively, under the alternative calibration methodology.

the model parameters using the new value for banks' share of financial assets $K_{b,t}$ while keeping other parameters unchanged. For the U.S., the parameter values for α, ω, μ, and σ become 0.17, 0.17, 0.71, and 0.81, respectively.

Appendix Figure 4 shows the steady state values of the key endogenous variables, NIM_t, N_t, $K_{b,t}/K_{h,t}$, x_t, p_t, and $\left(Q_t^* + Z_t^*\right) Q_{ss}$, for the U.S., under the new set of calibrated parameters, for various values of the target inflation rate $\overline{\pi}$. Quantitatively, the changes in the model parameters affect the recovery rate x_t considerably. For example, when the target inflation rate $\overline{\pi}$ is 0%, the recovery rate x_t is 1.04 under the alternative model, which is about the same value as that of the recovery rate in Japan under the baseline model. In addition, when the target inflation rate $\overline{\pi}$ falls to -3%, the recovery rate x_t approaches unity under the alternative model, while the recovery rate is well above unity in the baseline model. It is also important to note, however, that the implications of low nominal interest rates for bank profits and banking stability are not drastically different from what are obtained under the baseline simulation. In particular, while low nominal interest rates lower bank profits and undermine banking stability, the economy does not easily fall into a state where a run equilibrium exists unless the target inflation rate $\overline{\pi}$ takes an extreme value.[44]

[44] To economize on space, we do not provide figures for the other three countries. Generally speaking, for all three of the countries, the larger value for $K_{b,t}/K_{h,t}$ caused by the inclusion of financial assets held by "shadow banks" pushes the steady state recovery rate x_t downward, possibly due to the fact that the monitoring technology parameter α rises under the alternative calibrations. In contrast, bank profits NIM_t are hardly affected.

Figure 1: Interest Rates and Bank Profits



Notes: 1. For (b), all figures are taken from Holston, Laubach, and Williams (2017).

2. For (c), all figures are taken from Sudo, Okazaki, and Takizuka (2018).

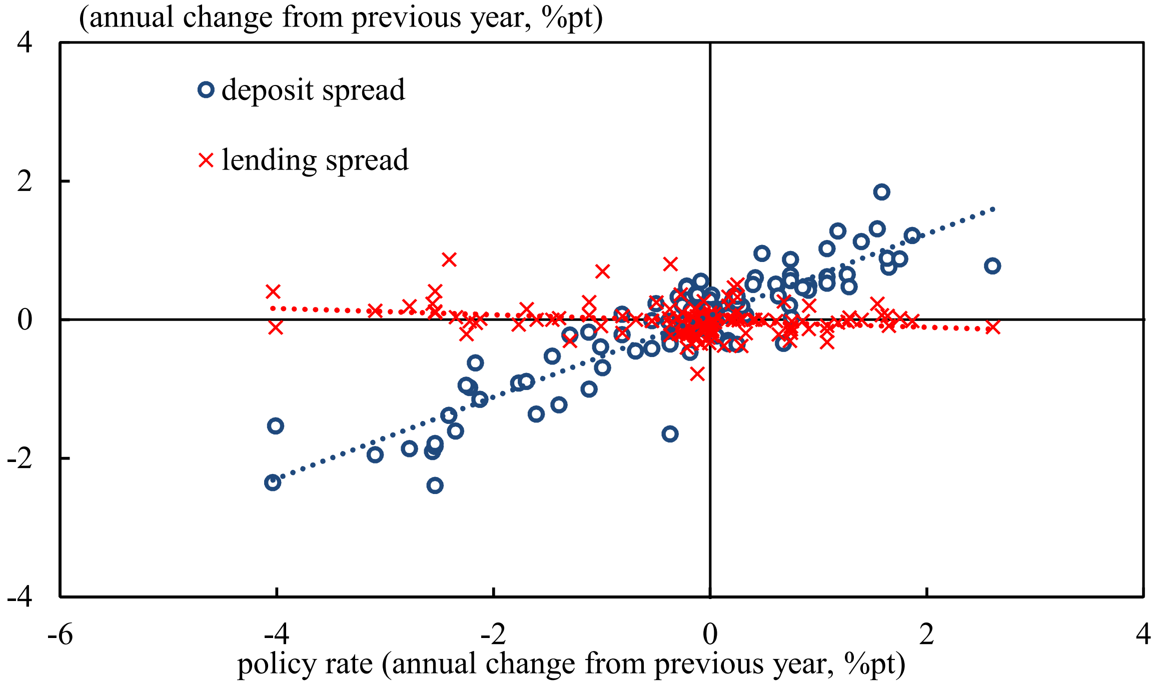
3. For (d), banks' loan-deposit interest spread is defined as "lending rate - deposit rate." The lending rate for Japan is the prime long-term interest rate; that for the U.S. is the prime loan rate; that for the U.K. is the average interest rate for "other loans to private non-financial corporations"; and that for Canada is the prime business loan rate. The deposit rate for Japan is calculated from aggregate income statement of banks compiled by the Japan Bankers Association; that for the U.S. is calculated from the Call report; that for the U.K. is calculated as the weighted average of households' sight and time deposit rates; that for Canada is calculated as the weighted average of households' savings and personal fixed-term deposit rates. The figures for Japan are based on the fiscal year.

4. For (d), the sample period for each country is as follows: from 1996 to 2017 for Japan; from 1985 to 2017 for US; from 2000 to 2017 for UK; from 1987 to 2017 for Canada; from 2004 to 2017 for Germany, France, and Italy. For Japan, the data before 1995 are dropped because deposit rates were regulated. See, for example, Itoh et. al. (2015) for details. For each of the other countries, the period is chosen based on the availability of the relevant data.

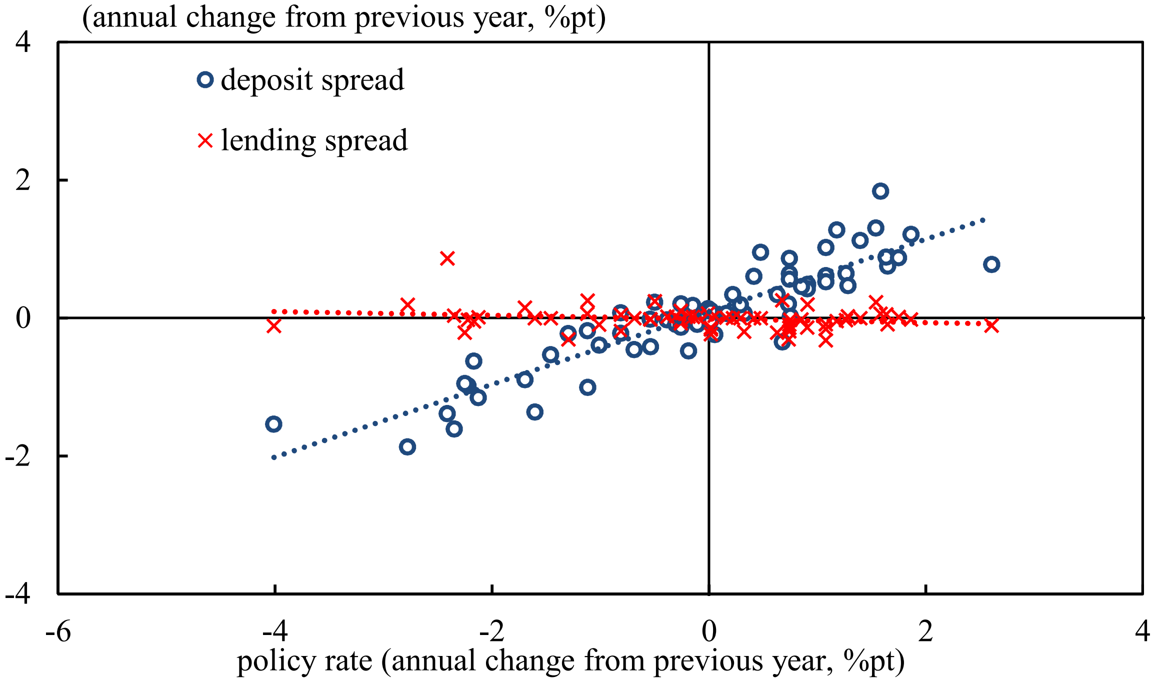
Sources: Bank of Canada (BOC), Bank of England (BOE), Bank of Japan (BOJ), FRED, FDIC, Holston, Laubach, and Williams (2017), Japan Bankers Association, Sudo, Okazaki, and Takizuka (2018).

Figure 2: Interest Rate Spreads and Nominal Interest Rates

(a) Correlation between Spreads and Nominal Interest Rates in G7 Countries, Including Post-crisis Period



(b) Correlation between Spreads and Nominal Interest Rates in G7 Countries, Excluding Post-crisis Period



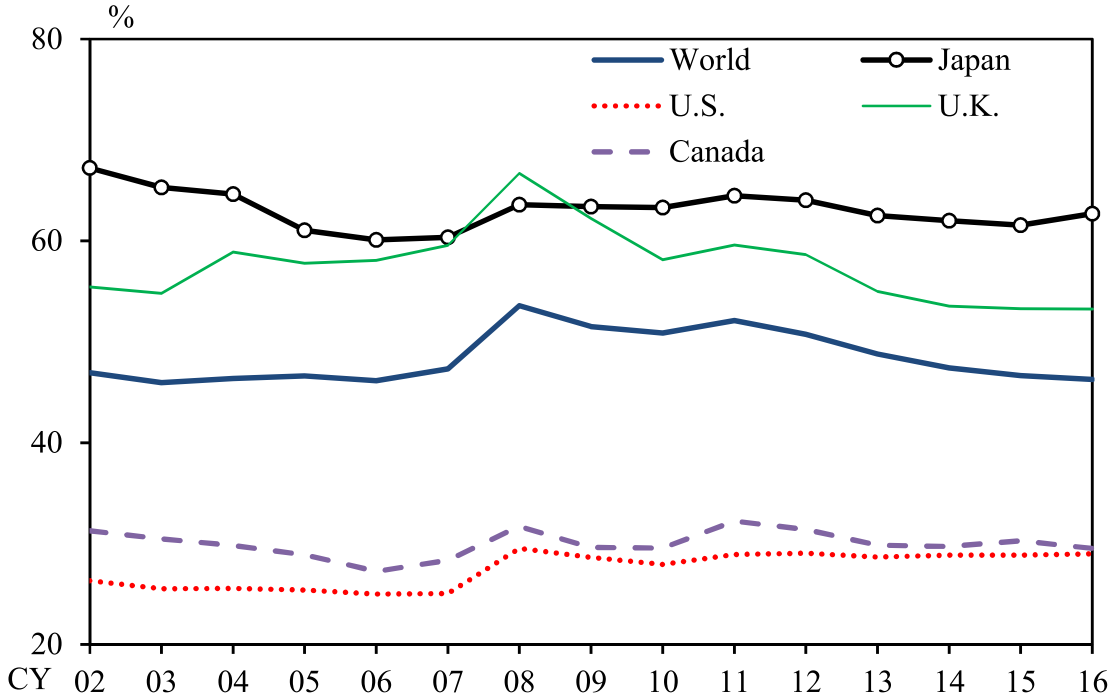
Notes: 1. The lending spread is defined as "lending rate - policy rate," and the deposit spread is defined as "policy rate - deposit rate". For Japan, the U.S., the U.K. and Canada, see the notes in Figure 1 for the definitions of interest rates. The lending rates for Germany, France and Italy are average loan rates. The deposit rates for these countries are average deposit rates.

2. For (a), see the notes in Figure 1 for the sample for each country. For (b), the entire sample for Japan and the sample period for the 2008 and beyond for the other countries are dropped.

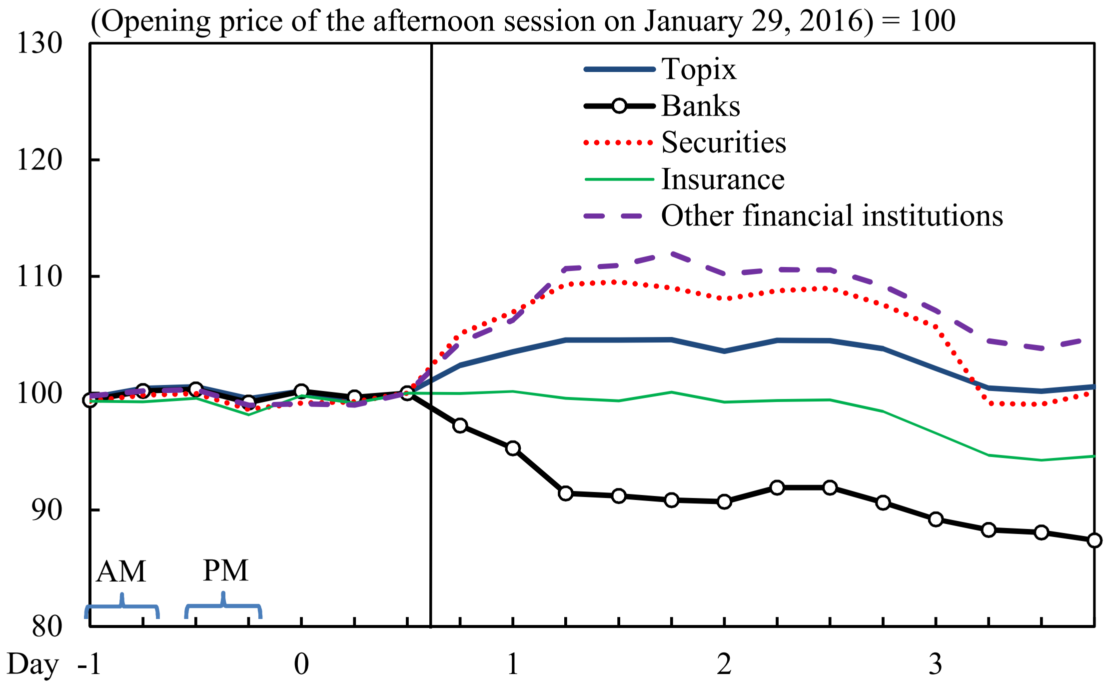
Sources: BOC, BOE, BOJ, ECB, FRED, FDIC, Japan Bankers Association.

Figure 3: Financial Transactions outside Banks

(a) Share of Financial Assets Held by Banks



(b) Stock Prices around January 29, 2016 in Japan



Notes: 1. For (a), the share of banks is defined as the share of assets held by banks among assets held by private financial corporations. Private financial corporations are defined as financial corporations excluding the central bank and public financial institutions.

2. For (b), the following prices for each day are shown: the opening price of the morning session; the closing price of the morning session; the opening price of the afternoon session; and the closing price of the afternoon session. The vertical line represents the timing of the announcement of the negative interest rate policy.

Sources: Bloomberg, FSB.

Figure 4: Outline of the Model

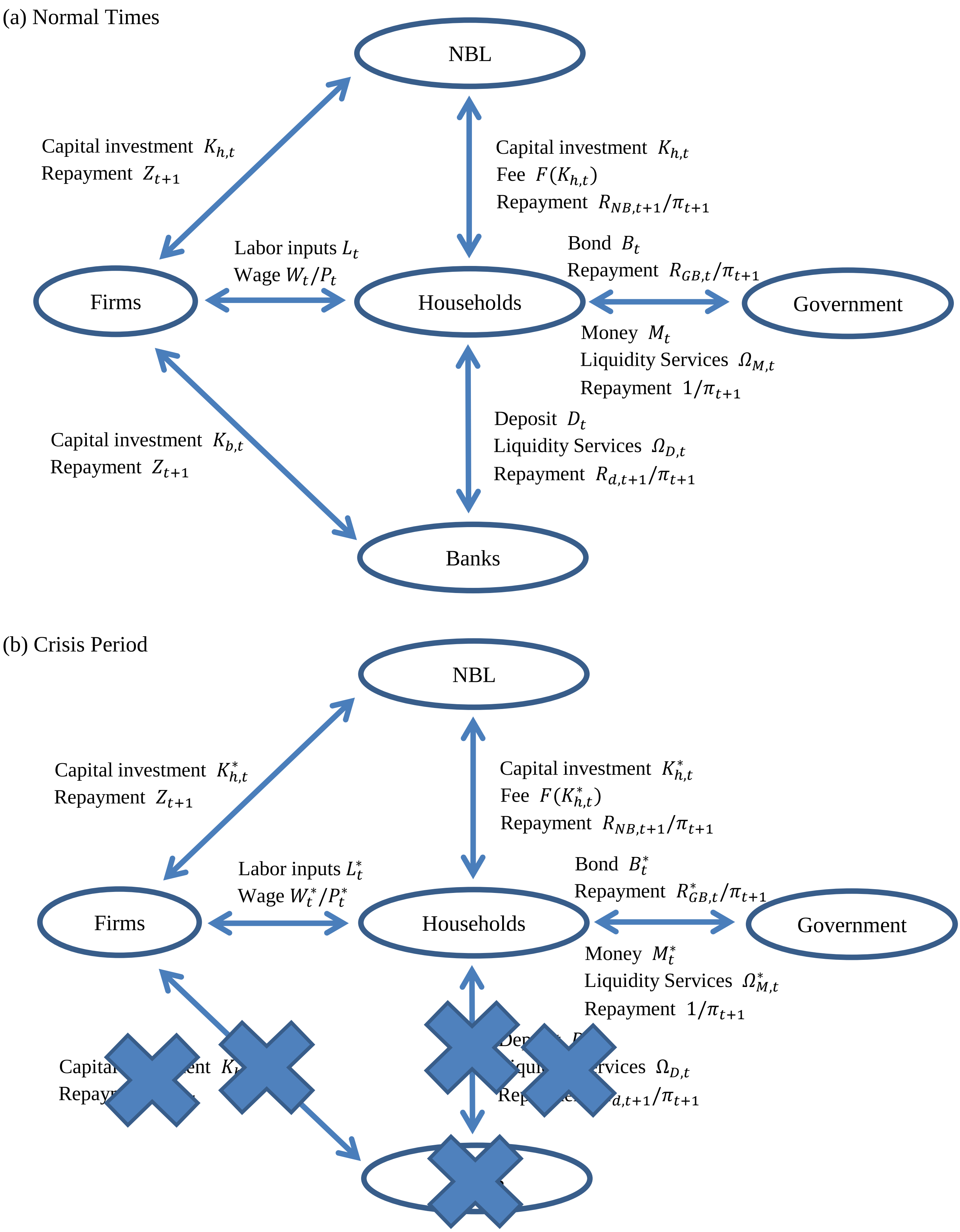
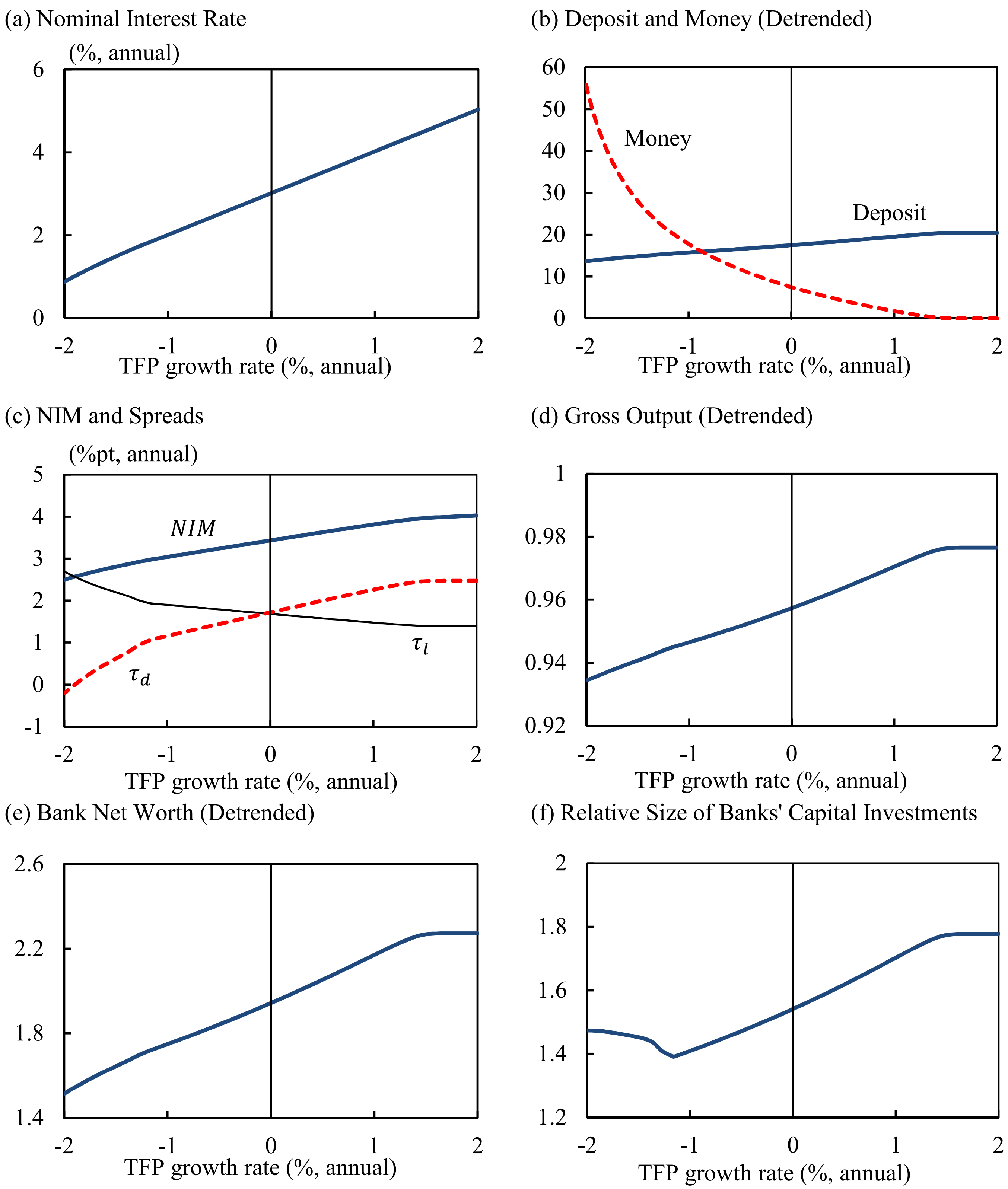


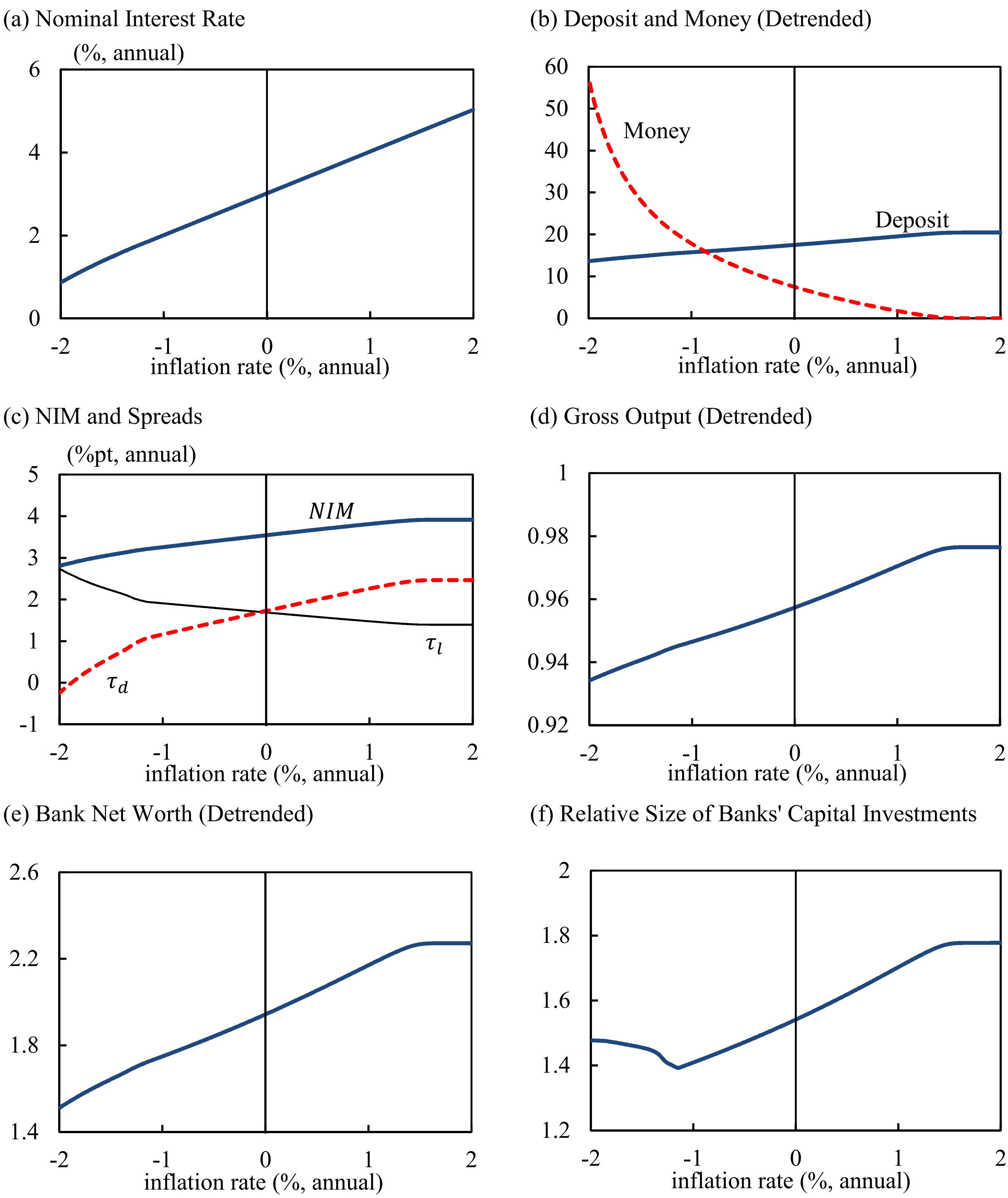
Figure 5: TFP Growth and Bank Profits



Notes: 1. The x-axis represents the TFP growth rate, and the y-axis represents the value of the corresponding variable at the steady state.

2. For (b), (d) and (e), the y-axis variables are detrended by an exponential TFP growth rate.

Figure 6: Inflation and Bank Profits

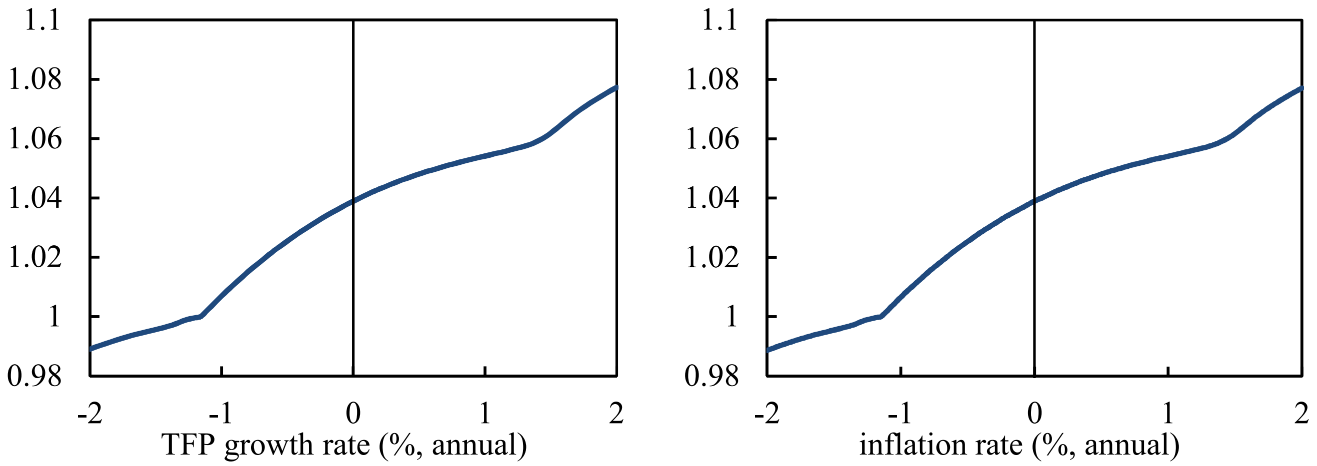


Notes: 1. The x-axis represents the target inflation rate, and the y-axis represents the value of the corresponding variable at the steady state.

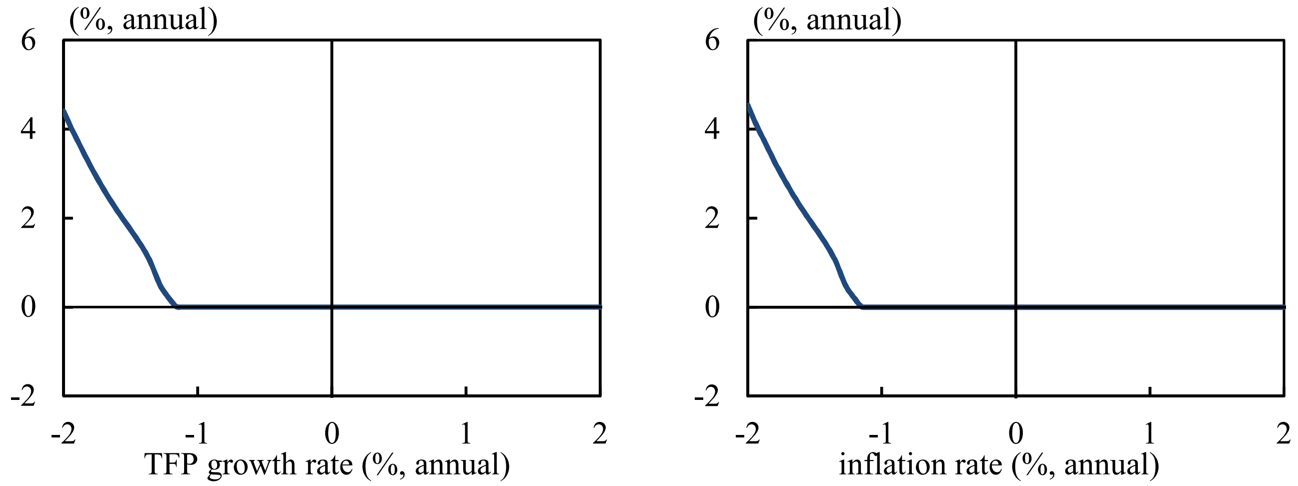
2. For (b), (d) and (e), the y-axis variables are detrended by an exponential TFP growth rate ($z = 1\%$).

Figure 7: Nominal Interest Rate and Banking Stability

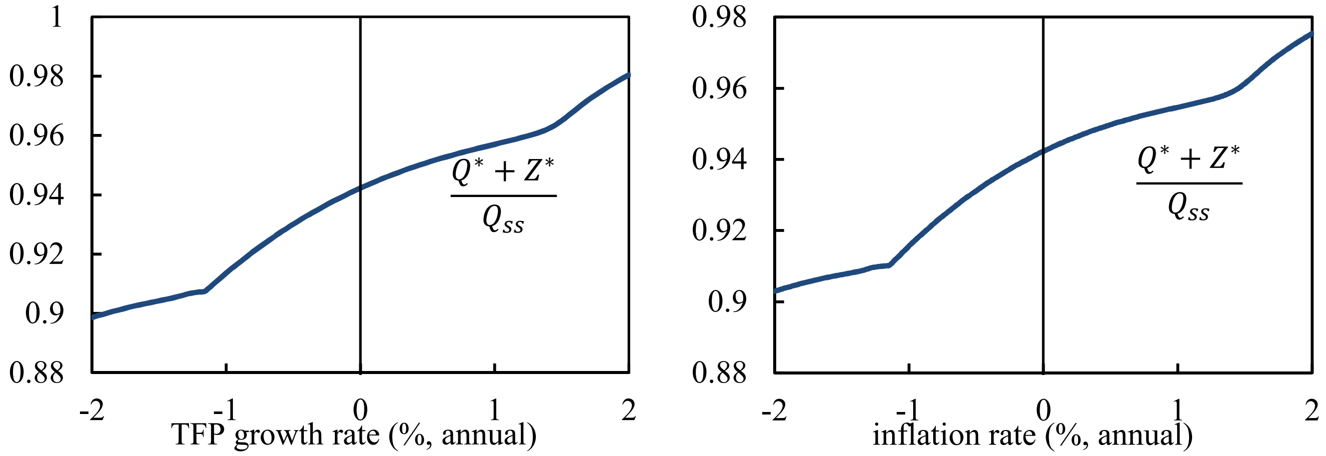
(a) Recovery Rate



(b) Probability of a Bank Run

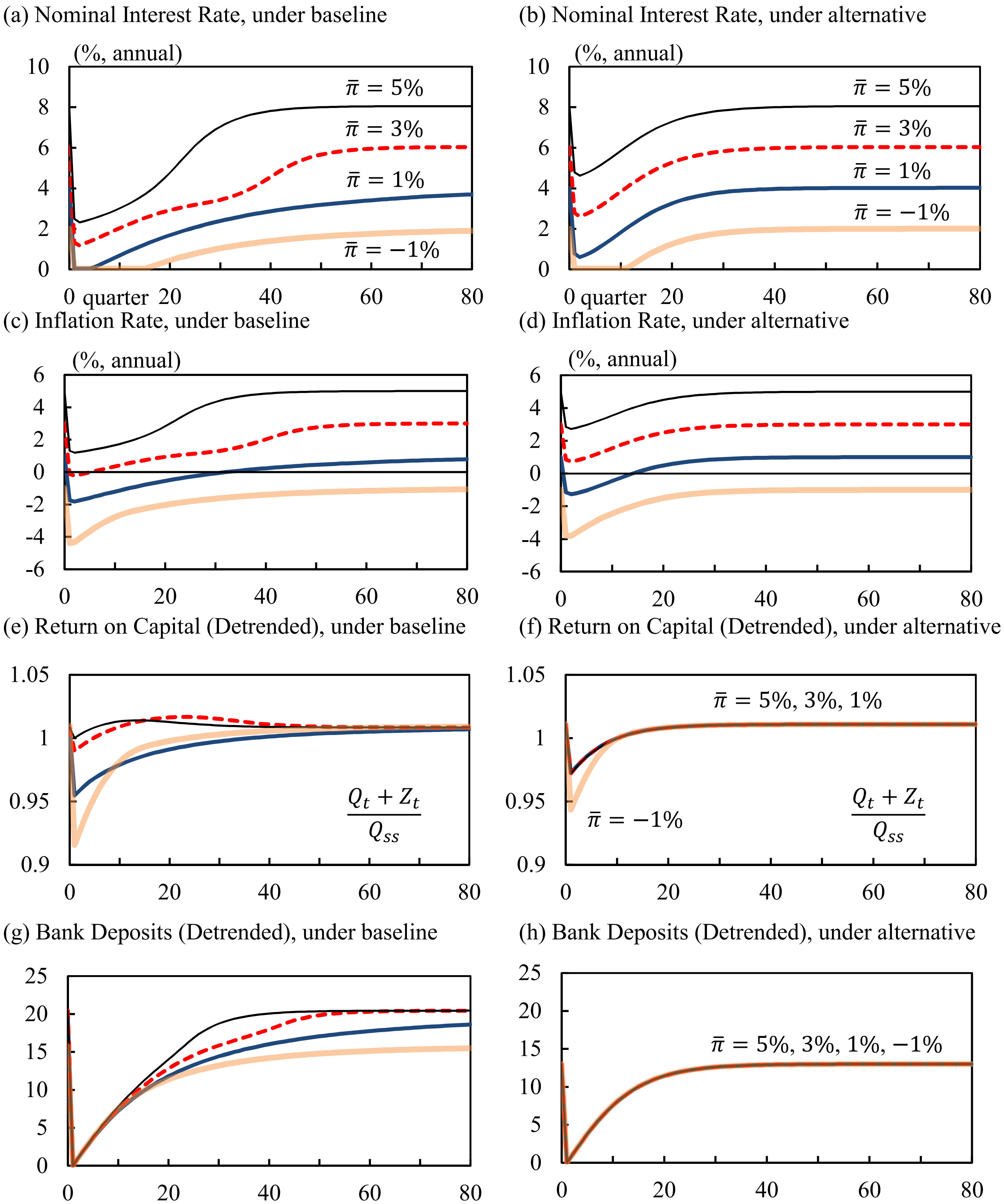


(c) Return on Capital during a Run State



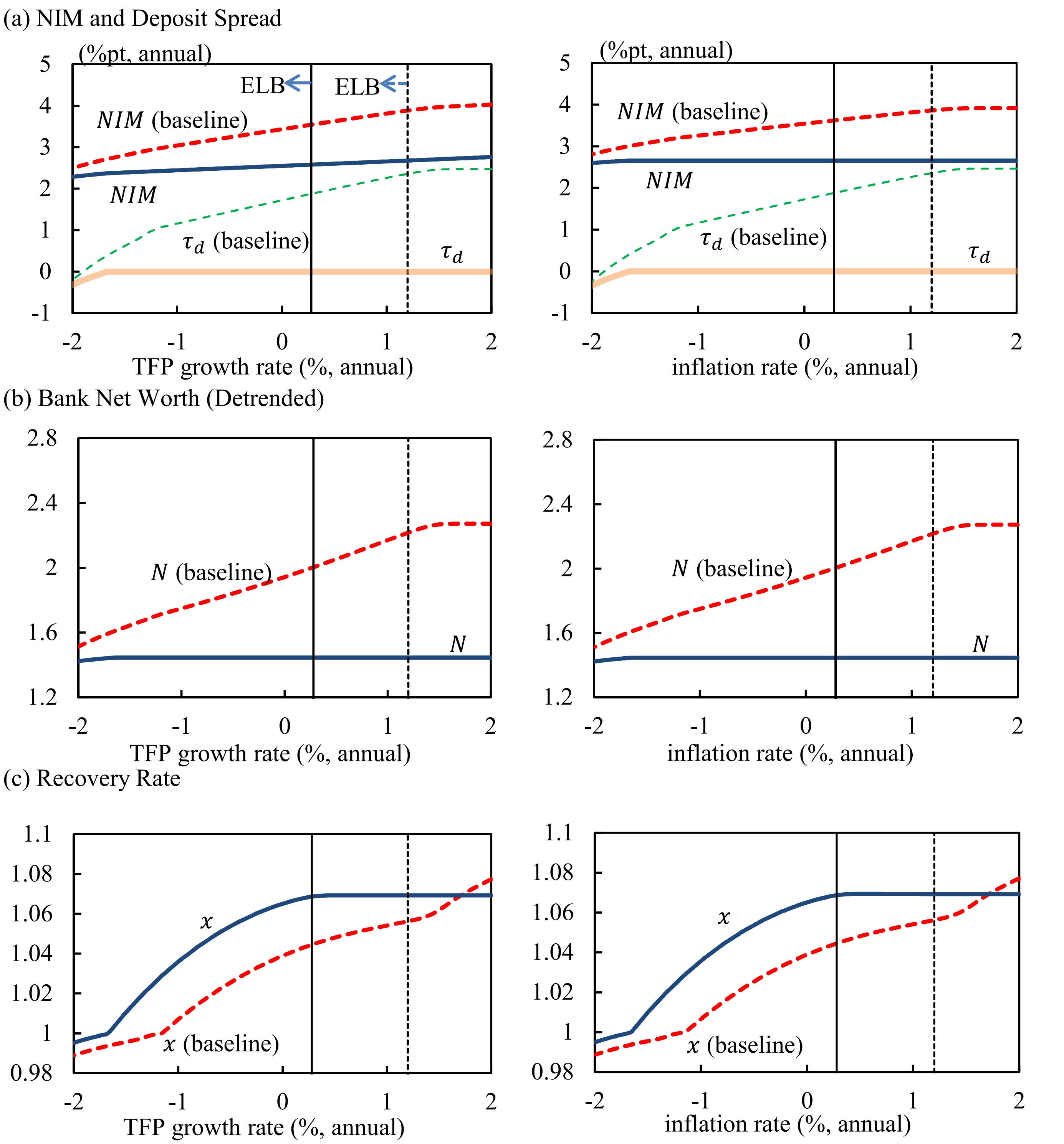
Note: For the left panels, the x-axis represents the TFP growth rate, and the y-axis represents the value of the corresponding variable at the steady state. For the right panels, the x-axis represents the target inflation rate, and the y-axis represents the value of the corresponding variable at the steady state.

Figure 8: Dynamics of the Economy following a Run



Notes: 1. (a), (c), (e), and (g) show dynamics of variables following a run under the baseline model.
(b), (d), (f), and (h) show dynamics of variables following a run under the alternative model.
2. For (e), (f), (g), and (h), y-axis variables are detrended by an exponential TFP growth rate ($z = 1\%$).

Figure 9: Nominal Interest Rates and Banking Stability under the Alternative Model

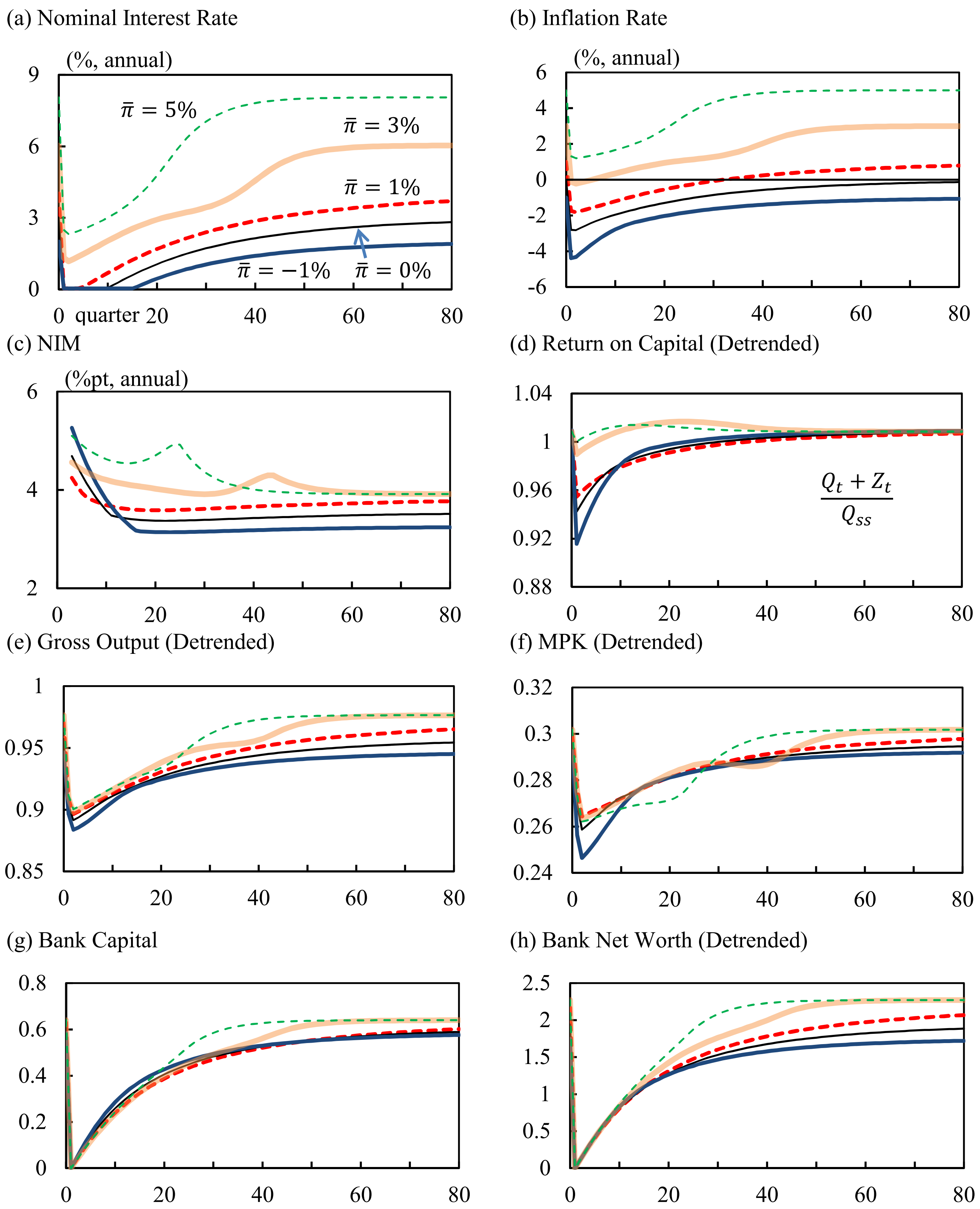


Notes: 1. For the left-hand side panels, the x-axis represents the TFP growth rate and the y-axis represents the value of the corresponding variable at the steady state. For the right-hand side panels, the x-axis represents the target inflation rate and the y-axis represents the value of the corresponding variable at the steady state.

2. The solid curves in each panel represent the figures based on the alternative model. The solid vertical line represents the threshold below which the ELB binds during a run in the alternative model. The dotted curves indicated as "baseline" represent the figures based on the baseline model. The dotted vertical line represents the threshold below which the ELB binds at during run in the baseline model.

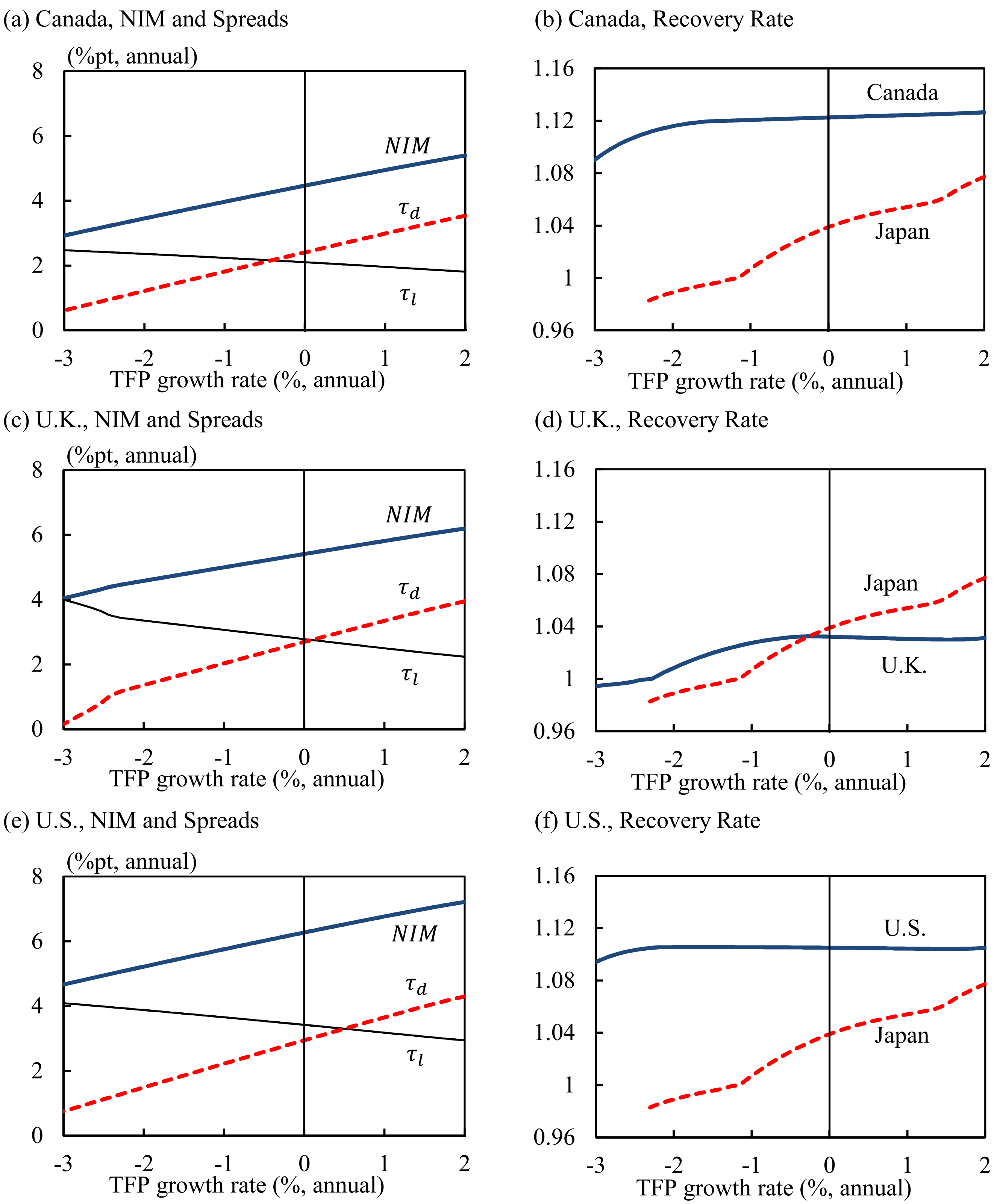
3. For the left-hand side panel of (b), the y-axis variable is detrended by an exponential TFP growth rate. For the right-hand panel of (b), the y-axis variable is detrended by an exponential TFP growth rate ($z = 1\%$).

Figure 10: Dynamics of the Economy following a Run



Note: For (d), (e), (f), and (h), y-axis variables are detrended by an exponential TFP growth rate ($z = 1\%$).

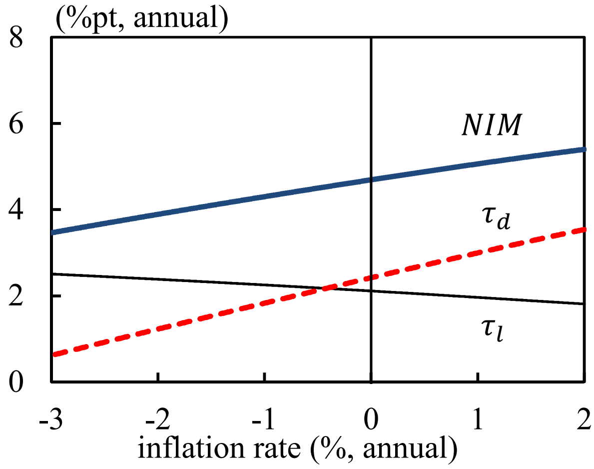
Figure 11: Effects of Nominal Interest Rates in Other Countries (1)



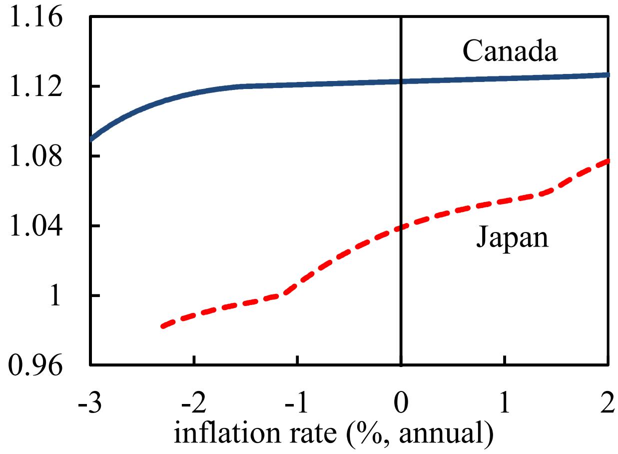
Note: The x-axis represents the TFP growth rate, and the y-axis represents the value of the corresponding variable at the steady state.

Figure 12: Effects of Nominal Interest Rates in Other Countries (2)

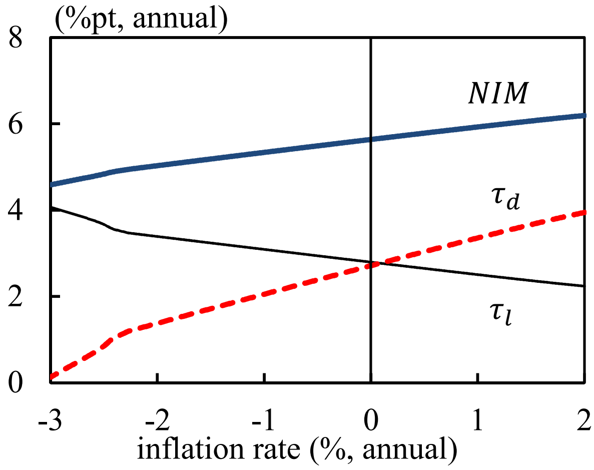
(a) Canada, NIM and Spreads



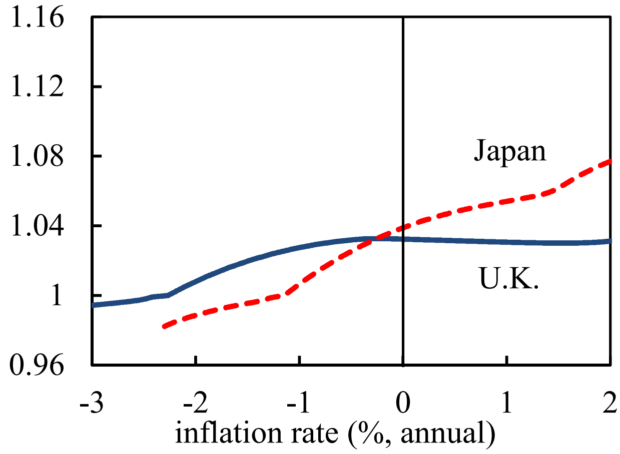
(b) Canada, Recovery Rate



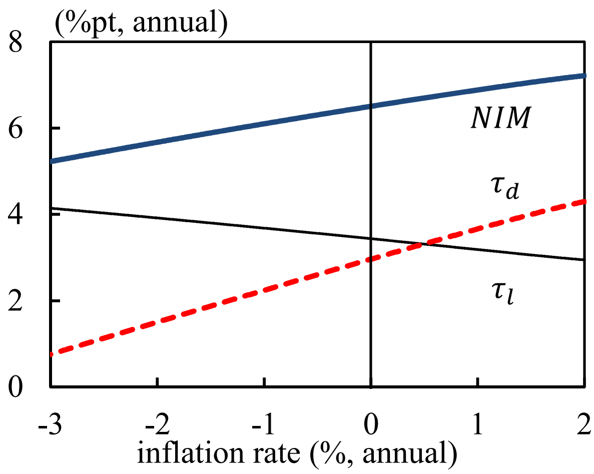
(c) U.K., NIM and Spreads



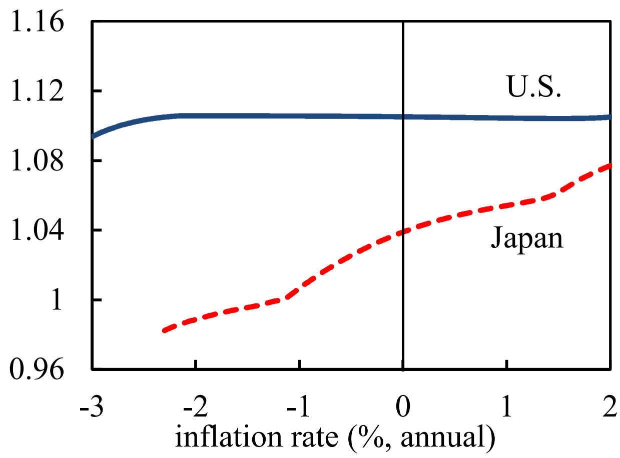
(d) U.K., Recovery Rate



(e) U.S., NIM and Spreads

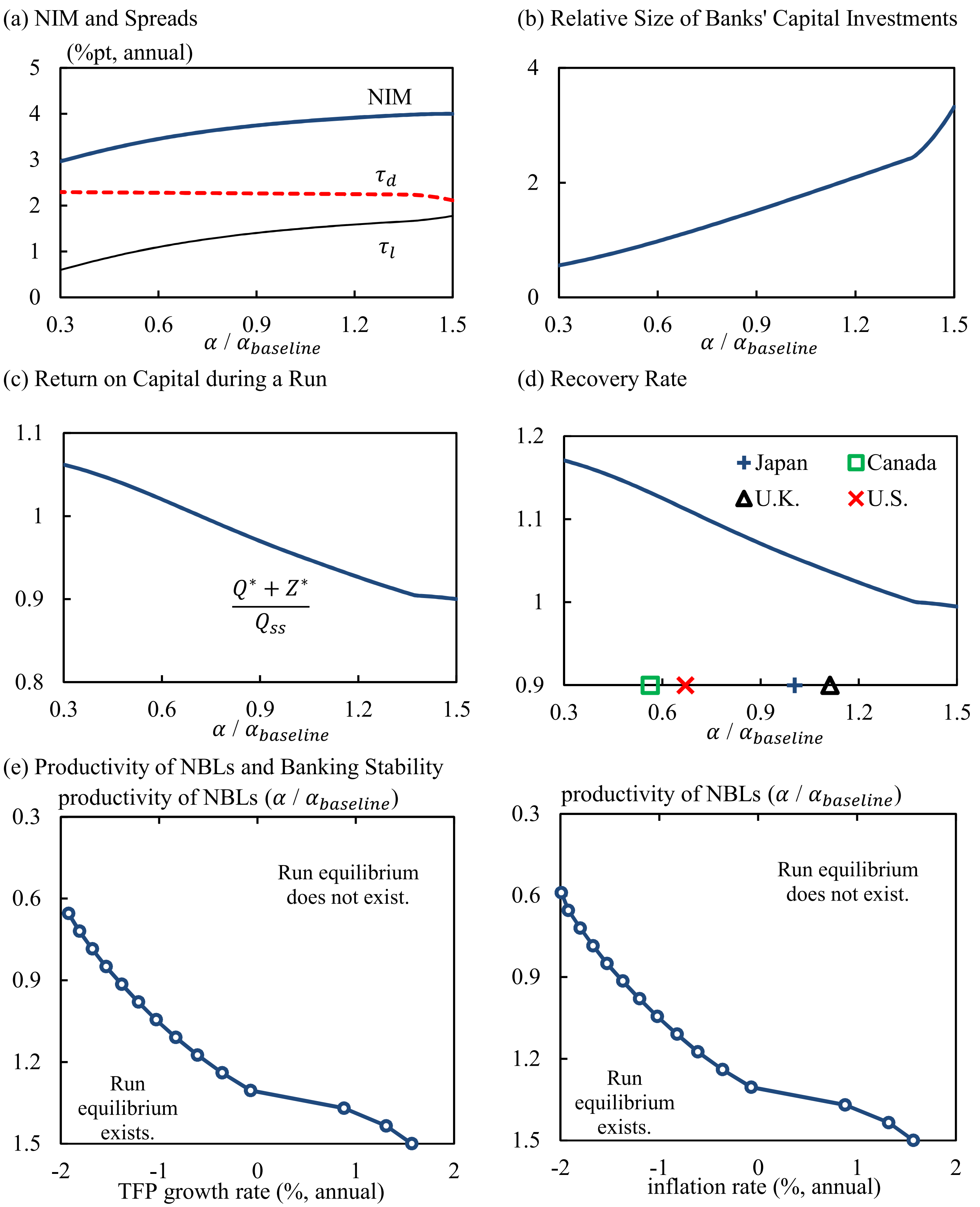


(f) U.S., Recovery Rate



Note: The x-axis represents the target inflation rate, and the y-axis represents the value of the corresponding variable at the steady state.

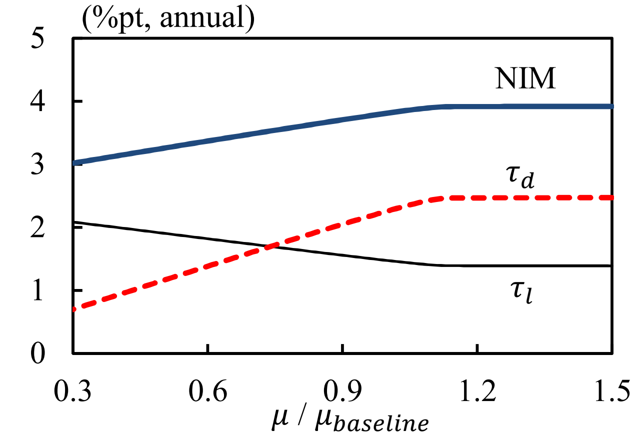
Figure 13: The Role of Productivity of NBLs



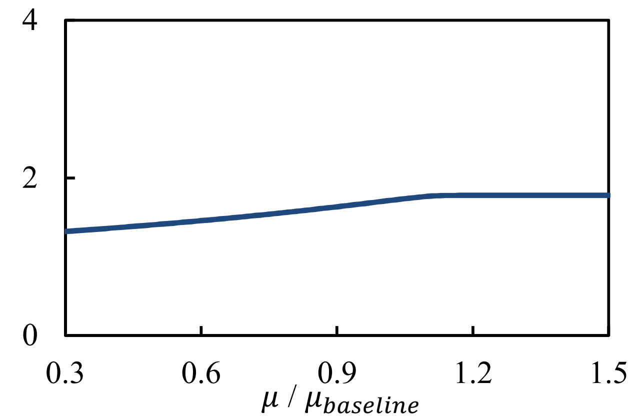
Notes: 1. For (a), (b), (c) and (d), the x-axis represents the asset management cost (normalized by its value in the baseline calibration), and the y-axis represents the value of the corresponding variable at the steady state.
2. For (d), the markers represent the calibrated value of the asset management cost for each country.

Figure 14: The Role of Liquidity of Deposits

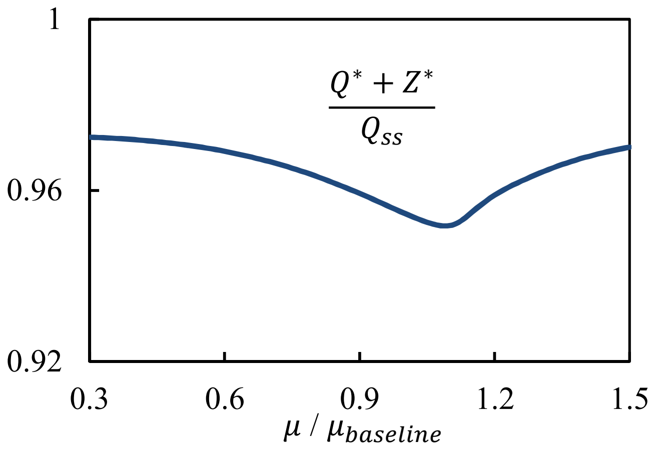
(a) NIM and Spreads



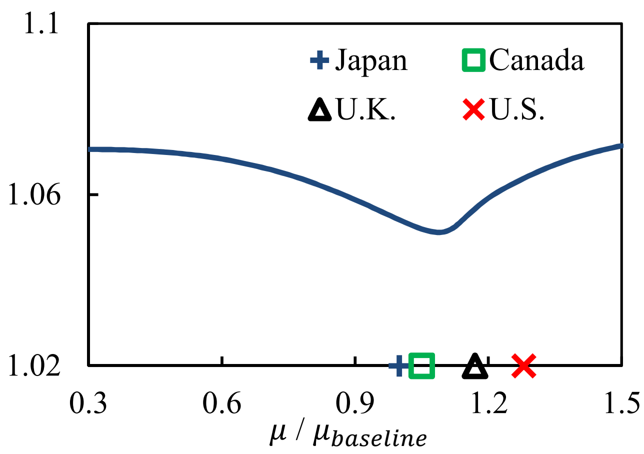
(b) Relative Size of Banks' Capital Investments



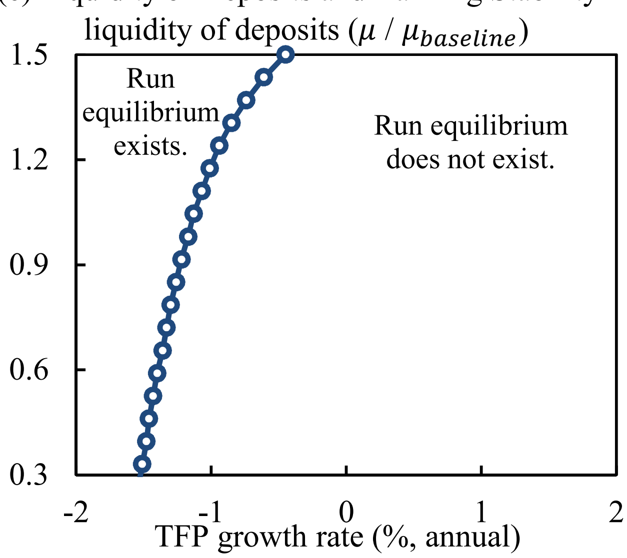
(c) Return on Capital during a Run State

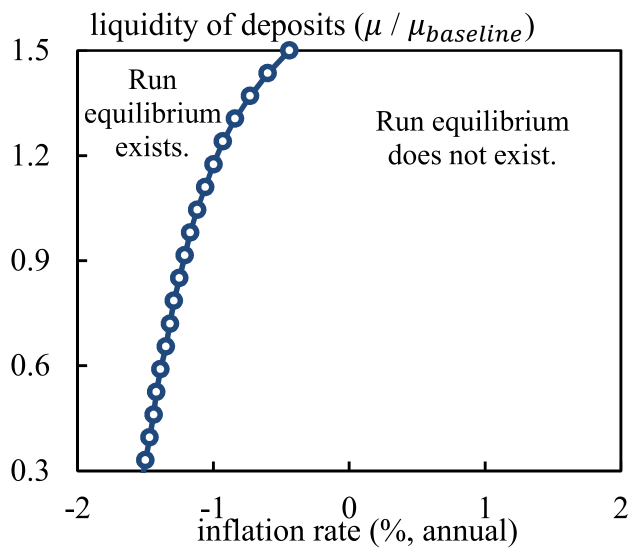


(d) Recovery Rate



(e) Liquidity of Deposits and Banking Stability

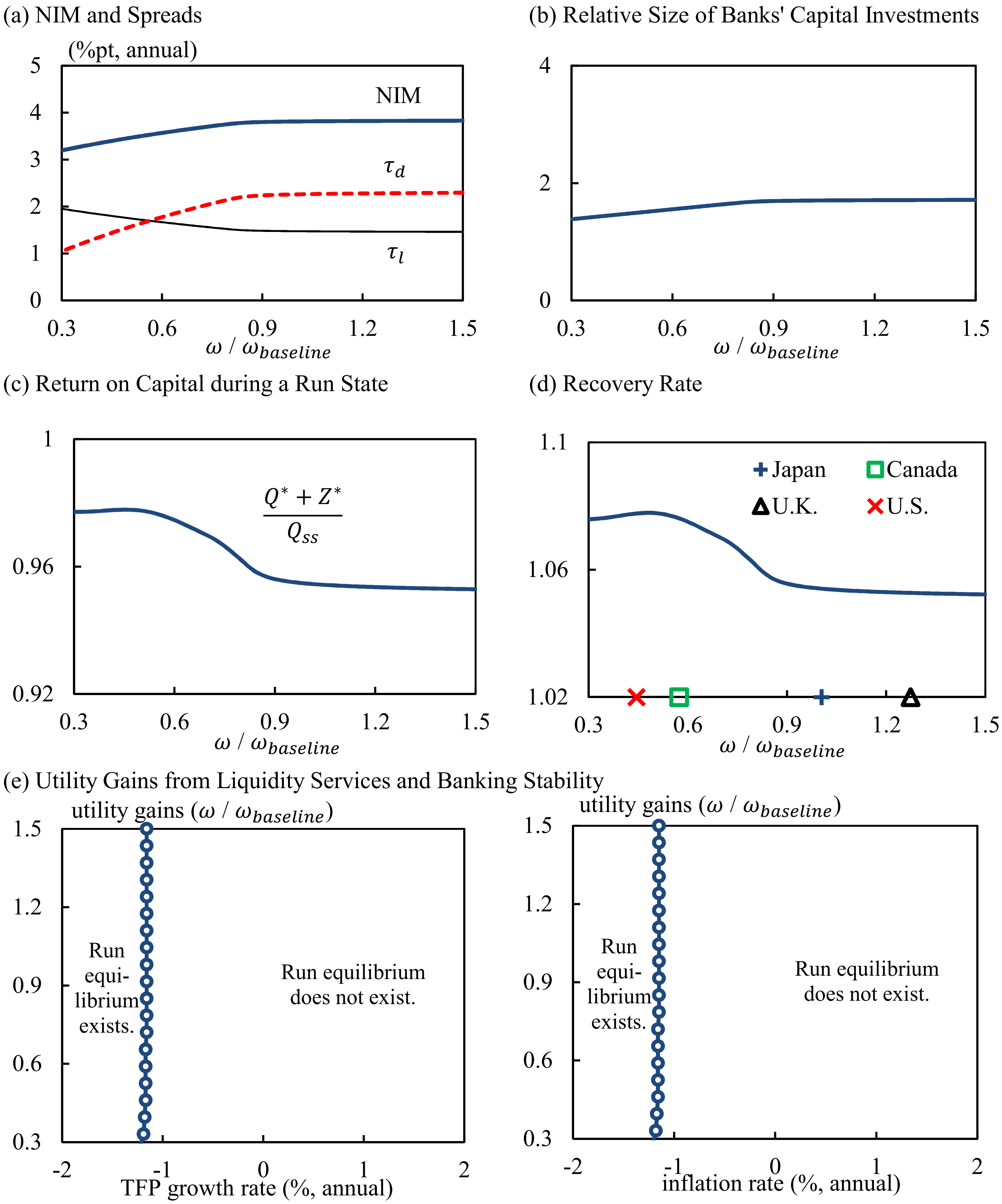




Notes: 1. For (a), (b), (c) and (d), the x-axis represents the relative size of liquidity services provided by deposits (normalized by its value in the baseline calibration), and the y-axis represents the value of the corresponding variable at the steady state.

2. For (d), the markers represent the calibrated value of the relative size of liquidity services provided by deposits for each country.

Figure 15: The Role of Utility Gains from Liquidity Services



Notes: 1. For (a), (b), (c) and (d), the x-axis represents the weight on utility from liquidity services by deposits and money holdings (normalized by its value in the baseline calibration), and the y-axis represents the value of the corresponding variable at the steady state.

2. For (d), the markers represent the calibrated value of the weight on utility from liquidity services by deposits and money holdings for each country.

Table 1: Correlation between Various Deposit Spreads and Nominal Interest Rates

Estimated equation:

$$\Delta(Deposit\ Spread)_t = \alpha + \beta\ \Delta(Nominal\ Interest\ Rate)_t + \varepsilon_t\ ,$$

where Δ denotes the difference from the previous year, and the deposit spread is defined by

$$Deposit\ Spread \equiv Nominal\ Interest\ Rate\ - Deposit\ Rate.$$

(a) Japan

choice of nominal interest rate	O/N rate	O/N rate	O/N rate	1-year Gov. Bond	1-year Gov. Bond	1-year Gov. Bond
choice of deposit rate	normal new	time new	average outstanding	normal new	time new	average outstanding
β	0.73** (0.04)	0.24** (0.07)	0.26 (0.16)	0.80** (0.03)	0.47** (0.09)	0.65** (0.17)
# of observations	22	22	22	22	22	22
estimation period	FY1996 - 2017					

(b) US

choice of nominal interest rate	FFR	FFR	FFR	FFR	1-year Treasury	1-year Treasury	1-year Treasury	1-year Treasury
choice of deposit rate	checking new	savings new	small time new	average outstanding	checking new	savings new	small time new	average outstanding
β	0.92** (0.02)	0.61** (0.05)	0.70** (0.08)	0.56** (0.03)	0.91** (0.03)	0.59** (0.06)	0.75** (0.11)	0.57** (0.05)
# of observations	27	27	27	33	27	27	27	33
estimation period	1987 - 2013	1987 - 2013	1987 - 2013	1985 - 2017	1987 - 2013	1987 - 2013	1987 - 2013	1985 - 2017

Notes: 1. We regress annual changes in the deposit spread on annual changes in the nominal interest rate, where deposit spread is defined as (nominal interest rate - deposit rate). The standard errors are in parentheses. ** and * denote statistical significance at the 5 % and 10 % levels, respectively.

2. The deposit rates for checking, savings and small time deposit accounts in the US are obtained from the supplementary data of Drechsler, Savov, and Schnabl (2017).

Sources: BOJ, Drechsler, Savov, and Schnable (2017), FDIC, FRED, Japan Bankers Association.

Table 2: Calibrated Parameters for Japan

(a) Calibrated Parameters

	Parameter	Value
β	Subjective discount factor (quarterly)	0.995
χ	Weight on disutility arising from labor inputs	1
ν	Inverse of Frisch elasticity	1
η	Labor Share	0.66
ε	Elasticity of substitution between differentiated wholesale goods	11
κ	Parameter that governs the size of the cost of changing prices of wholesale goods	1,000
A	Technology level	1.0
z	Growth rate of TFP (annual)	0.01
ϕ	Leverage of banks	10
ϕ_π	Policy weight attached to inflation rate	1.5
δ	Parameter that governs the size of the disutility of holding deposits at a run	3
ζ	Elasticity of substitution between money and deposits	0.99
α	Parameter that governs the size of asset management costs	0.17
ω	Weight on utility arising from liquidity services by deposits and money holdings	0.19
μ	Relative size of liquidity services provided by deposits	0.57
σ	Probability that a banker survives until the next period	0.87
w_b	Endowment for new bankers	0.83
$\bar{\pi}$	Target inflation rate (annual)	1.01

(b) Targets

	Variable	Value
$\frac{\Delta\tau_d}{\Delta R_b}$	Elasticity of deposit spread with respect to nominal interest rate	0.5
τ_l	Lending spread (%pt, annual)	1.48
$\frac{M}{D}$	Ratio of households' holdings of money and deposit	0.089
$\frac{K_b}{K_h}$	Relative size of banks' capital investment	1.70

Table 3: Calibrated Parameters for the Other Countries

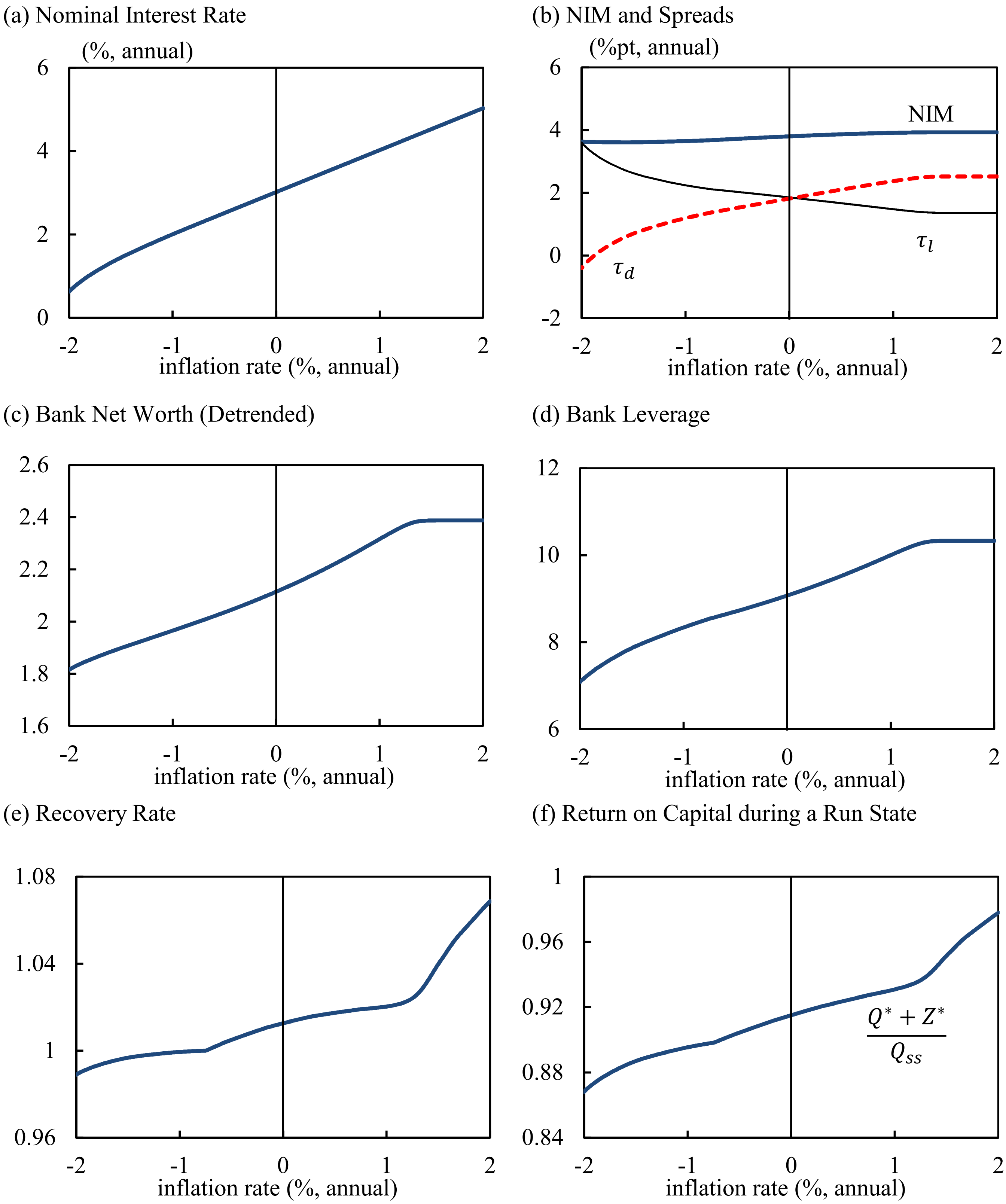
(a) Calibrated Parameters

Parameter	Canada	U.K.	U.S.	Japan
β	0.995	0.995	0.995	0.995
χ	1	1	1	1
ν	1	1	1	1
η	0.66	0.66	0.66	0.66
ε	11	11	11	11
κ	1,000	1,000	1,000	1,000
A	1.0	1.0	1.0	1.0
z	0.02	0.02	0.02	0.01
ϕ	10	10	10	10
ϕ_π	1.5	1.5	1.5	1.5
δ	3	3	3	3
ζ	0.99	0.99	0.99	0.99
α	0.10	0.19	0.11	0.17
ω	0.11	0.25	0.09	0.19
μ	0.60	0.67	0.74	0.57
σ	0.84	0.83	0.81	0.87
w_b	0.28	0.48	0.16	0.83
$\bar{\pi}$	1.02	1.02	1.02	1.01

(b) Targets

	Canada	U.K.	U.S.	Japan
$\frac{\Delta\tau_d}{\Delta R_b}$	0.5	0.5	0.5	0.5
τ_l	1.82	2.25	2.96	1.48
$\frac{M}{D}$	0.080	0.047	0.038	0.089
$\frac{K_b}{K_h}$	0.43	1.38	0.37	1.70

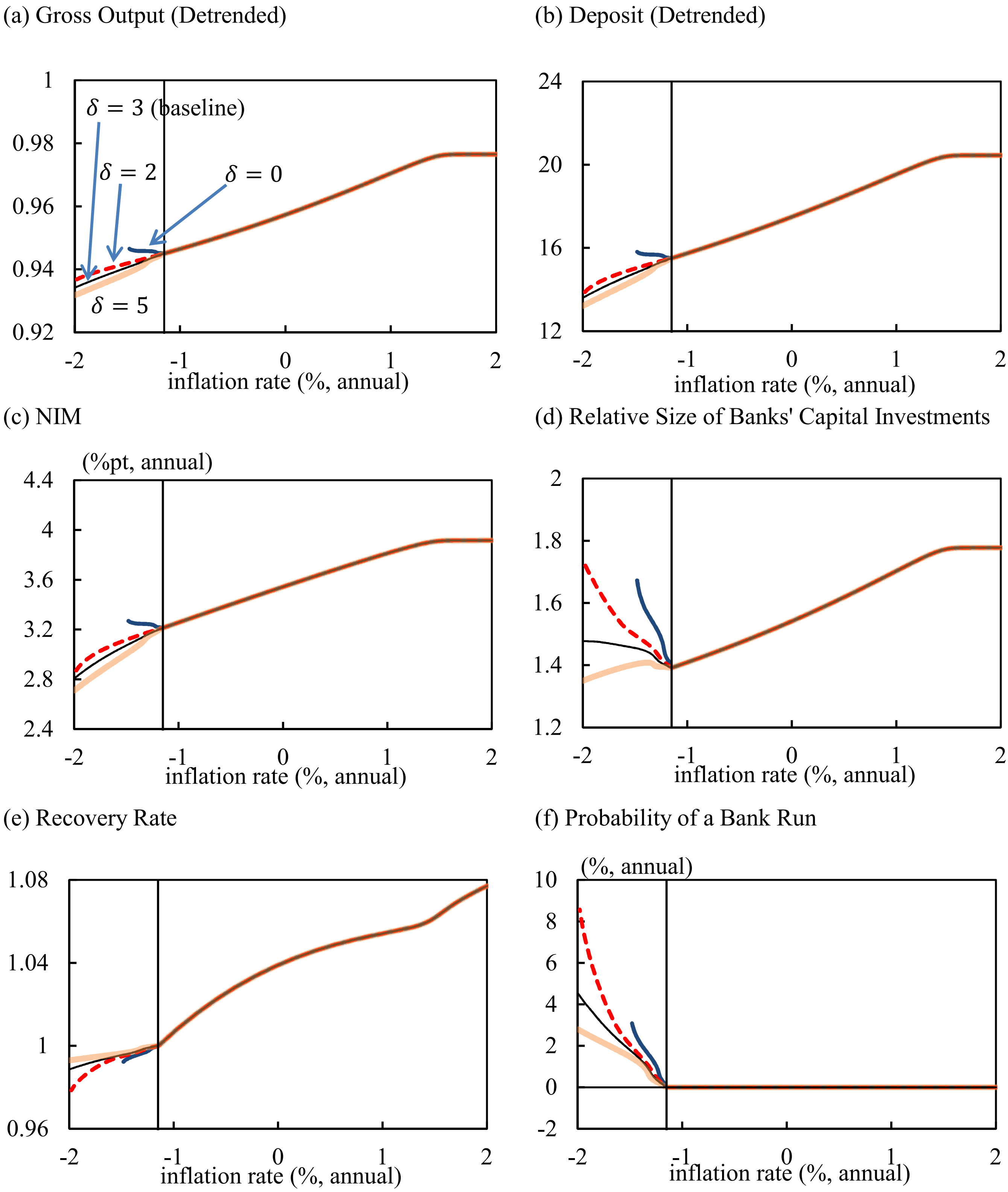
Appendix Figure 1: Effects of Inflation Rate under GK model



Notes: 1. The x-axis represents the target inflation rate, and the y-axis represents the value of the corresponding variable at the steady state.

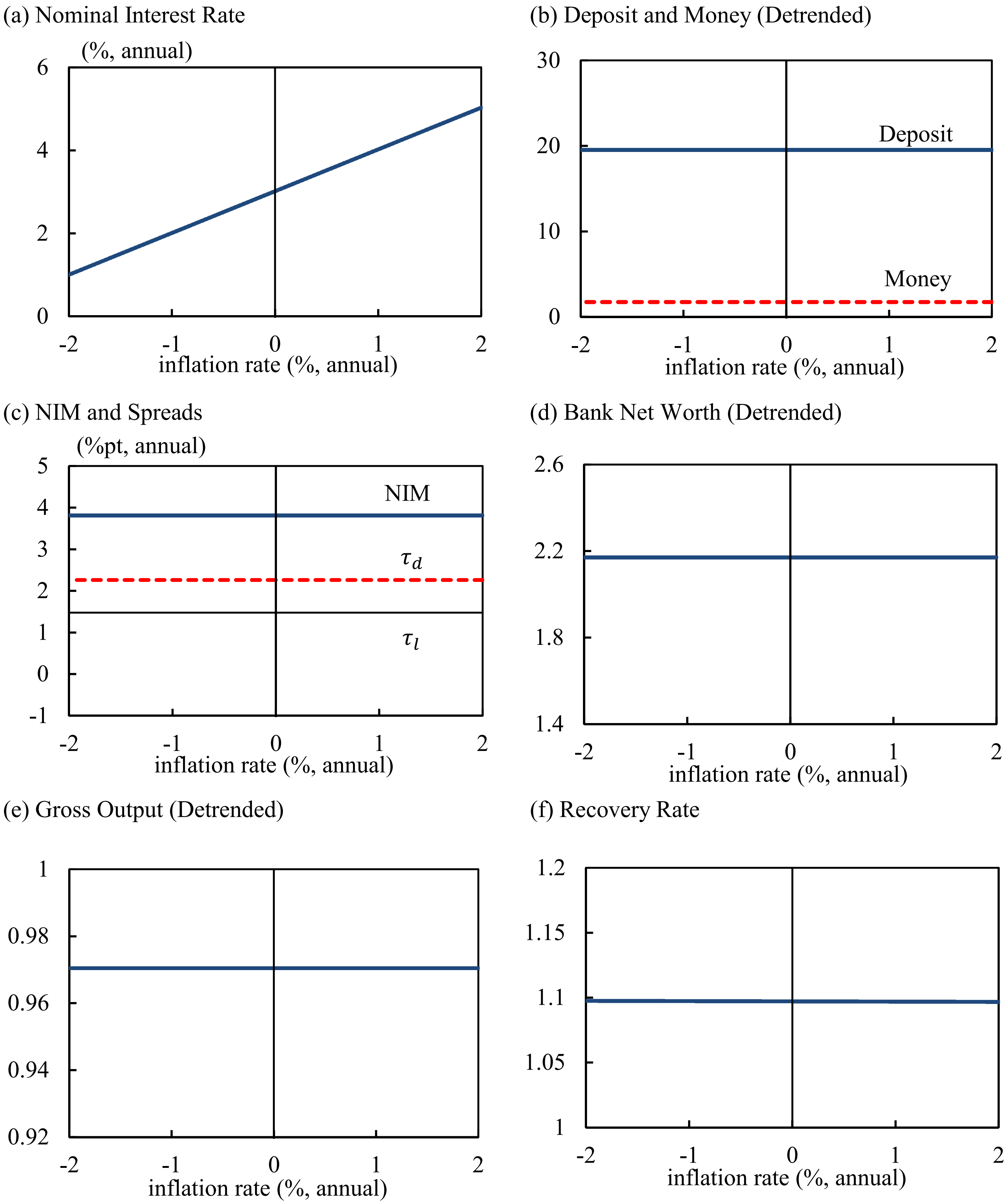
2. For (c), the y-axis variables are detrended by an exponential TFP growth rate ($z = 1\%$).

Appendix Figure 2: Implications of Disutility of Deposit Holding during a Run



Notes: 1. The x-axis represents the target inflation rate, and the y-axis represents the value of the corresponding variable at the steady state.
2. For (a) and (b), the y-axis variables are detrended by an exponential TFP growth rate ($z = 1\%$).
3. The vertical line in each panel represents the highest value of the target inflation rate during which a run equilibrium exists in the steady state.

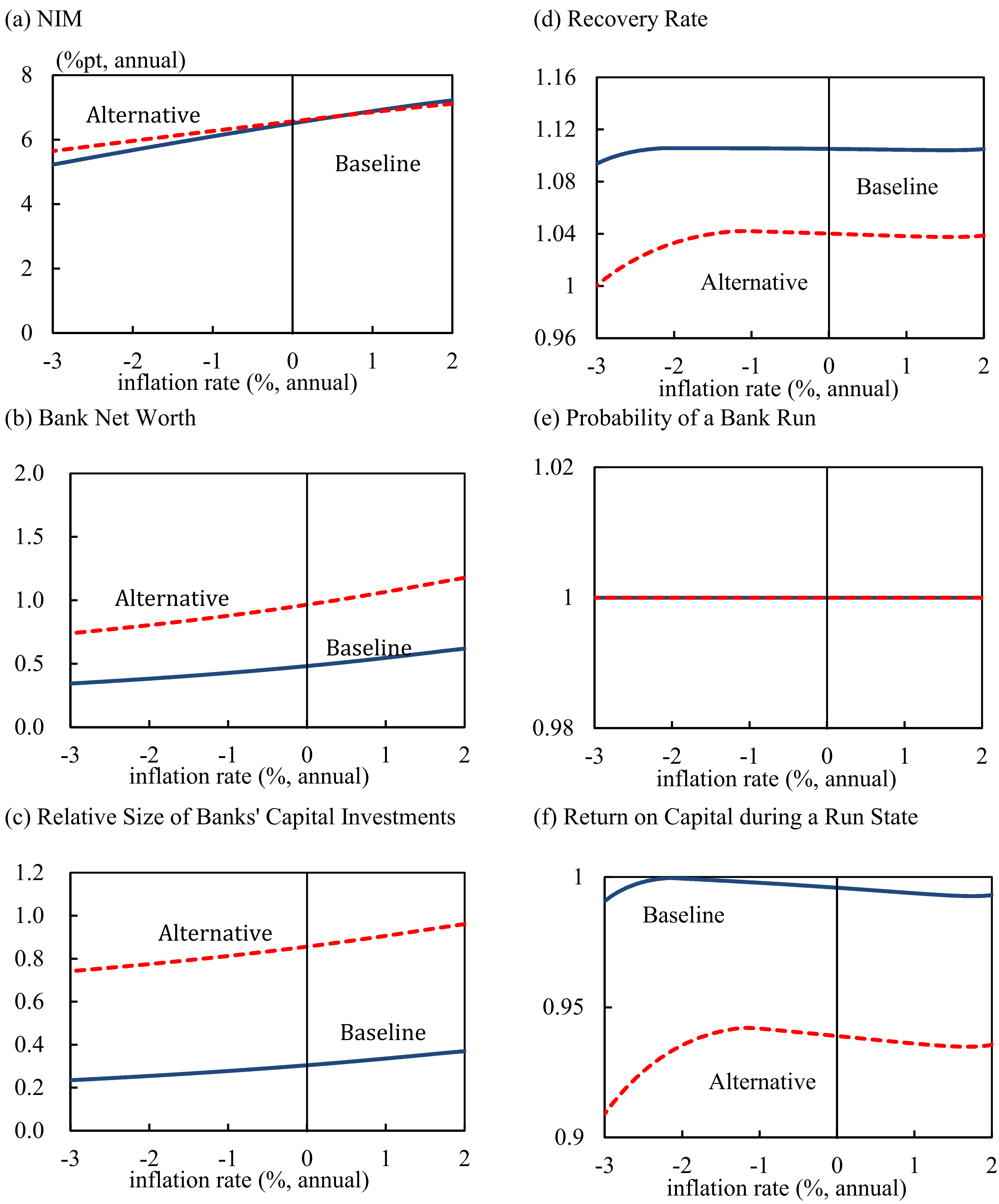
Appendix Figure 3: Effects of Inflation Rate under the Model with Interest Rate on Money



Notes: 1. The x-axis represents the target inflation rate, and the y-axis represents the value of the corresponding variable at the steady state.

2. For (b), (d) and (e), the y-axis variables are detrended by an exponential TFP growth rate (z = 1%).

Appendix Figure 4: Effects of Nominal Interest Rates under an Alternative Definition of Banks (for the U.S. only)



Note: The x-axis represents the target inflation rate, and the y-axis represents the value of the corresponding variable at the steady state.